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VOLUME XIV NUMBER 5

THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

LXXV

MAY–JUNE 1902

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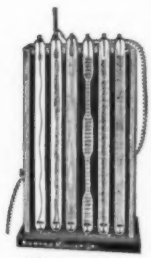

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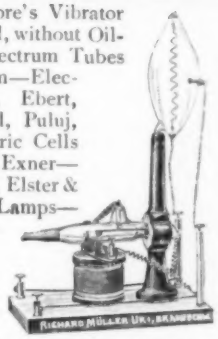

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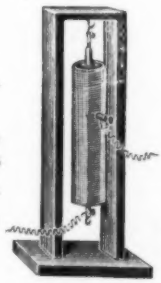

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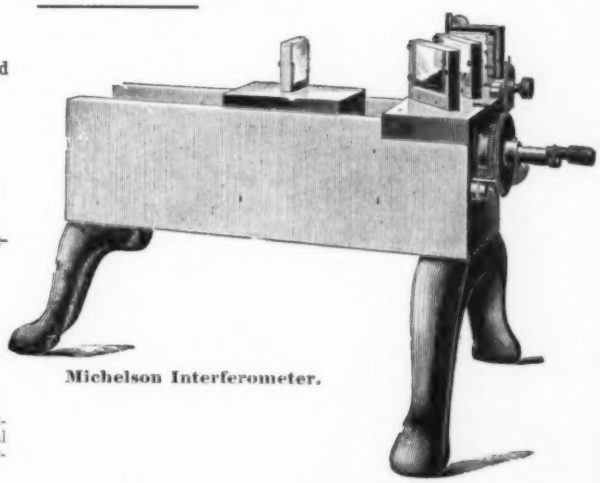

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THE

PHYSICAL REVIEW.

THE VALUE OF THE GRAVITATION CONSTANT.

By G. K. Burgess.

IN 1885 Harkness[1] made a calculation by the method of least squares of the value of the gravitation constant, but the best experimental work has been done since then, and it seems desirable to-day to deduce from all the data at hand the most probable value of the gravitation constant and its analogue the mean density of the earth. Boys, in his report to the Paris Congress, 1900, has made a critical study of the various methods, but does not suggest a best value for the constants. To do this last is our object.

The gravitation constant is the quantity K in the formula which expresses the Newtonian law of attraction:

$$F = K\frac{mm'}{r^2}.$$

This constant is related to the expression for the mean density of the earth, approximately as follows:

$$\Delta = \tfrac{3}{4}\cdot\frac{g}{Kr}$$

in which g is the acceleration due to gravity of a mass at the surface of the earth whose radius is r and mean density is Δ.

We may classify the methods for measuring K and Δ in two categories: Astronomical or geodetic, as those employing the

[1] Harkness, Append. III., Washington Obs., 1885. Bibliography in Recherches sur la Constante de Gravitation, Burgess, 1901; pub. by Hermann, Paris.

plumb-line, the pendulum and any method depending upon the variation of the acceleration of gravity; and the methods employing some form of the torsion balance, the chemical balance and the metronome pendulum.

For the first, use is made of a considerable extent of ground and includes a geological survey of the rocks in the neighborhood. In the second, measurements are made in the laboratory upon known masses and distances.

The oldest measures belong, in spite of the fact that the work of Cavendish dates from the eighteenth century, to the first category. In this first group quantities are dealt with which have always been very uncertain—uncertainties of the same order as those involving the investigation of the density of the outer strata of the earth.

In the second category, all the accuracy that may be obtained is subordinate to measurements of very small length and sometimes of time, and to adjustments for symmetry.

Let us examine these various methods for the determination of our constants. We will first consider the observations based on astronomical or geodetic methods. It was Bouguer who first tried to establish such a determination by measuring the attraction of a mountain. Maskelyne and Hutton in 1775 were the first to obtain results by this method.

Two processes exist for the determination of the attraction of a mountain: In the first, the attraction of the mountain on a plumb-line is measured by joining a point on the north to a point on the south by triangulation. The latitudes of these points are determined astronomically, and their positions, compared to those furnished by the triangulation, give the value of the deviation of the plumb-line.

The second process, as does the first, requires a study of the rocks that compose the mountain and a determination of the diminution of the acceleration of gravity at the summit of the mountain as compared with measurements made at the base reduced to sea level.

This same principle of the variation of the acceleration due to gravity is applicable to measurements made in mines, as was first done by Airy.

Another method depending upon the attraction of great terrestrial masses was proposed by Thomson and Tait. This would be to

measure the variation of g at the seashore where there are very high tides as in the Bay of Fundy.

Coming to what we may call the indoor methods, Wilsing made use of a metronome pendulum of very long period, swinging at first alone and then beside a very heavy mass. This method is susceptible of more accuracy than is given by the ordinary use of a pendulum.

Such scientists as Von Jolly, Poynting, Koenig, Richarz and Menzel have employed the chemical balance for this determination. Von Jolly's apparatus was installed in a tower 21 meters high. He measured the change in weight produced in a spherical mass of mercury of 5 kg. by a lead mass of 5,775 kg. placed directly beneath, while another 5 kg. mass remained in the other balance pan at the top of the tower. This method necessitates a knowledge of the variation of the acceleration of gravity for the place of observation—a variation that is never in accord with the theoretical calculations—also changes of temperature played a most important vitiating rôle.

Poynting made two series of determinations of which the second is an improvement over the first. He had a long-arm balance of considerable sensibility which he further increased by the use of a mirror suspended by two threads: the one fixed, the other attached to the arm of the balance. The masses could be hung at two levels to eliminate the attraction of the balance-arm. The attracting mass was placed below the balance on a turn-table which a fourth smaller mass kept from tilting. The suspended masses weighed 21.5 kg. and the attracting mass 153 kg. Two independent determinations made with the same apparatus gave him 5.52 and 5.46 for Δ. The balance-bëam rested continuously upon its knife-edge during a series; the temperature variations were a minimum.

Recently, Richarz and Krigar-Menzel have used the chemical balance in the following way: A rectangular mass of lead of 100,000 kg. carried a balance to which could be suspended, either above or below the lead mass, mercury spheres of 1 kg. hung from the balance-beam which was was 23 cm. long with a half-period of ¾ min. It was necessary as in Jolly's case to determine the variations of g with the height, both with and without the presence of

the lead mass. The change in weight was determined by displacing the mercury spheres alternately from the upper to the lower positions. To obtain a precision of 0.1 per cent. it would be necessary to measure a weight of 0.0001 mg.

The order of magnitude of the quantities measured was 1 mg. The greatest pains were taken to obviate suspected sources of error, but the balance was raised and lowered from its knife-edge during a series.

A balance that turns about a vertical axis may be rendered more sensible than a balance oscillating about a horizontal axis. It is for this reason that Cavendish's balance and its various modifications have been the most often used to determine the gravitation constant. This method possesses still another advantage: that of giving us the value of this constant independently of the local value of the acceleration of gravity. Besides here we do not have to guard against dust whose effects may be appreciable in Richarz's method. In the torsion method, the most serious sources of error arise from uncertainties caused by variations of temperature which give rise to convection currents, and to the imperfect elasticity of the suspension wire.

The classic set-up for the method is the following: Two small masses are hung from the extremities of a horizontal beam supported by as fine a wire as possible. Two heavy masses attract the others. The deflection produced in the zero position of the movable system by a known change in position of the heavy masses is sought. This angle, the geometrical data, the time of oscillation, the moment of inertia and the value of g being known, Δ and k may be computed.

Cavendish made his investigations in 1793 and although Baily, with a large subsidy from the British Government, took up the matter in 1841, the result of his two thousand observations is of a much less importance than that of Cavendish, who showed the important rôle played by small changes of temperature in the apparatus and its vicinity. In 1873 Cornu and Baille determined the influences of a resisting medium on a torsion balance and showed that the resistance of the air for very small velocities is rigorously proportional to the velocity. These scientists also showed the advantage in re-

ducing the dimensions of the apparatus, that by this its sensibility was in nowise reduced, rendering also its behavior more constant. They were the first to make use of a chronograph for measurements of this nature.

Boys, who commenced his measurements in 1889, made an apparatus of almost microscopic dimensions. Cavendish's balance-arm was 180 cm. long, Cornu's 50 cm., but Boys' only 2.3 cm. The masses were proportionally reduced. In the Cavendish apparatus the attracting spheres were 30 cm. in diameter, while those in Boys' apparatus were but 6 cm. in diameter. His maximum deflection was about one degree, the couple to be measured 200 times as small as that of Cornu's and 1,500 times as small as Cavendish's. It is probable that Boys went too far with such very small dimensions, for it may happen that the errors arising from the geometrical measurements and from want of symmetry play a considerable part. In discovering how to make quartz fibers and their elastic properties he acquired precious auxiliaries, the non-homogeneity of the suspension wire having always been a source of great uncertainty. In order to increase still more the sensitiveness of the system, Boys placed his two pairs of spheres at different levels, thus avoiding the attraction in opposite sense of each of the large upon the more distant small sphere. An advantage gained by the smallness of dimensions in this apparatus was to render minimum the perturbations produced by air currents due to convection or temperature changes.

Doctor Carl Braun is the first who has succeeded in operating his apparatus in vacuo. The attracting masses are of mercury of 900 grams each, placed at a distance of 40 cm. The attracted masses weigh 54 grams and are attached at either end of an aluminium arm of 24.6 cm. length. To obtain the results already published he made use of a brass wire which he is at present replacing by quartz to continue his researches, in which he operates by two methods: the classic method of deflection and the method of oscillation, which consists in noting the time of oscillation for the positions of maximum and minimum attraction of the heavy masses. Instead of using a scale and telescope placed at several meters from the apparatus, his telescope is but a few decimeters distant. The

measurements of deflection are made by means of a scale of a few centimeters length and an ocular, and the luminous beam undergoes several reflections and refractions which may introduce indeterminable errors of considerable size.

The balance of Eötvös can be turned horizontally as a whole and the measurements of the time of oscillation for the maximum and minimum positions between two quadrangular pillars which are the attracting masses are then taken. The sensibility of this apparatus is such that it is capable of detecting the variations in the horizontal component of gravity at a point distant only 50 cm. from another. A quartz fiber is not used.

SUMMARY OF EXPERIMENTAL RESULTS.

1. *Astronomical and geodetic methods.*

Observer.	Station.	Result.	Date.
Maskelyne and Hutton.	Schehallen, Scotland.	4.71 and 4.45	1775
Carlini.	Mt. Cenis, Italy.	4.77, 4.95, 4.84	1821
Airy.	Harton, England.	6.57 and 5.48	1854
James and Clarke.	Arthur's Seat, Scotland.	5.32	1855
Pechmann.	Gerold, Alps.	6.13	1865
Mendenhall.	Fuji-yama, Japan.	5.77	1880
Sterneck.	Pibram, Bohemia.	4.77	1883
	Freiberg, Saxony.	6.77	1885
Preston.	Haleakala, S. I.	5.57	1887
	Mauna Kea.	5.13	1892

Mean : 5.60 ± 0.13, $p = 0$.

(Without Sterneck's second value.)

2. *Wilsing's pendulum* 5.5579 ± 0.018, $p = 1$ (1889).

3. *Observations with chemical balance.*

Observer.	Result.	Date.
V. Jolly.	5.692 ± 0.068	1881
Poynting.	$5.493[\pm 0.03]$	1891
Richarz and Krigar-Menzel.	5.505 ± 0.009	1898

Mean : 5.507 ± 0.014, $p = 2$.

4. *Observations with tension balance.*

Observer.	Result.	Date.
Cavendish.	5.45 [± 0.05]	1798
Reich.	5.49 ± 0.023	1837
	5.583 ± 0.015	1852
Baily.	5.67 or 5.55	1843
	[5.55 ± 0.05]	
Cornu and Baille.	5.50 and 5.56	1878
	[5.54 ± 0.02]	
Boys.	5.5270 ± 0.0019	1895
	[5.5226 ± 0.0041]	
Braun.	5.5273 ± 0.0012	1896
Eötvös.	5.53 [± 0.010]	1896

Mean: 5.5243 ± 0.0009, $p = 300$.

The mean, deduced from all these series, treated by the method of least squares, is:

$$J = 5.5247 \pm 0.0013.^{1}$$

The corresponding value of the gravitation constant is:

$$K = 666.07 \cdot 10^{-10} \quad \overline{\text{cm.}^3}/\text{gr.}\ \overline{\text{sec.}^2} \pm 0.16 \cdot 10^{-10}.$$

Let us consider the above series:

Series I.—There is great uncertainty in the value found for the mean density by these methods for reasons already indicated, and the mean of all these measurements has a probable error of 4 per cent. In fact, it would seem futile to attempt exact measurements by such methods, by reason of the uncertainty of quantities, such as the density of mountains and the superficial strata of the earth, which enter into the equations. Also, no great progress has been made here since the first attempts.

Series III.—There are but three determinations by this method but their respective weights increase greatly from the first to the last. It is probable that the figure found by Richarz and Krigar-Menzel is as accurate as can be hoped to be obtained by our chemical balances of to-day. As in the first series, the problem is complicated by the variation of g with elevation. Richarz found a difference of 0.5 per cent. in this variation for his two determina-

[1] The probable error is indicated.

tions at an interval of two years—introducing a like uncertainty into his results.

Series IV.—We have here a considerable number of results which are closely grouped about the mean. The quantities in brackets are the estimated values and probable errors. Baily made a constant error in the interpretation of his results; the corrected value is given. In Boys' case the value in brackets is the result deduced from all of his series and not from the four highest as he deduced it. The most recent observations by this method—those of Boys and Braun—are practically identical with each other and with the mean deduced from all observations.

If we consider the results of the last two series for the last twelve years, we see that the chemical balance gives a value 0.5 per cent. lower than that obtained by the torsion balance for the mean density. Nevertheless, taking all things into account I think it may safely be said that we know the mean density of the earth and the gravitation constant to well within 0.001 part of their respective values, as indicated above.

If the weight assigned to the torsion-balance determinations be reduced from 300 to 3, we have:

1. Wilsing's pendulum.....	$\Delta = 5.5579$	$p = 1$
2. Chemical balance.....	5.507	$p = 2$
3. Torsion balance.....	5.5243	$p = 3$
Mean.....	$\Delta = 5.5241$	

This result is identical with the previous one.

It would seem desirable to make a new determination by Wilsing's method.

UNIVERSITY OF CALIFORNIA,
February, 1902.

THE VELOCITY OF IONS FROM HOT PLATINUM WIRES. II.

By C. D. Child.

THE second part of this article deals principally with the discharge from a hot platinum wire in other gases than air and in a vacuum. The work is not at all a complete investigation, but I shall not be able to continue it further at present, and therefore publish the experiments which have already been performed.

But first I wish to describe a few experiments with the particles driven from the wire. The presence about the wire of particles which diminish the velocity of ions has already been shown. It was noticed that the copper wires to which the platinum was attached became oxidized and that there was thus a possibility that particles of copper oxide were producing the effect. The copper wire was therefore removed and only platinum wire was used in the enclosed tube. The decrease in the amount of discharge was then even more marked than before.

A second test regarding the same point was then made by heating the copper wire and then examining the air about it for the presence of nuclei in the formation of clouds. Such nuclei were indeed found when the copper was sufficiently heated, but the amount of current which had been used in former experiments was not sufficient to heat the copper which was used, so as to give off such particles. This seems to be sufficient to show that those which produce the decrease in the rate of discharge came from the platinum itself.

The effect of these particles on the ions coming from a flame was examined. Several turns of platinum wire were placed between pieces of wood about 1.5 cm. apart, and 10 cm. long. The air arising from these wires was passed between two pieces of tin of the same length and the same distance apart and 6 cm. in width. These were kept at a potential difference of a hundred volts. They

were for the purpose of taking any ions out of the air arising from the platinum. The air was then allowed to pass between a flame kept at 50 volts and a circular plate 5 cm. in diameter surrounded by a guard ring. The plate was about 2 cm. from the flame.

A charged plate was first put in the place of the flame and no discharge was noted when the wire beneath was heated. This showed that the pieces of tin were properly serving their purpose of drawing the ions out of the ascending air.

The effect of the ascending air on the discharge from the flame was then noted. When the flame was charged positively and the wire beneath was not heated, the rate of discharge was 8×10^{-10} ampère. When the wire was heated it was 4×10^{-10} ampère, showing a decrease of 50 per cent. caused by the particles arising from the platinum. When the flame was discharged negatively, the rate of discharge was 12×10^{-10} ampère, and with the wire heated 3.2×10^{-10} ampère, a decrease of 83 per cent. This experiment was modified in various ways, but the decrease in the velocity of the negative ions was always greater than that of the positive.

It seems most reasonable that this should be so. Discharge occurs from positive platinum wires at a lower temperature than from a negative one. Apparently there is greater attraction between the platinum and negative ions than between platinum and positive ions. If this be true, we must expect a greater attraction between the particles of platinum given off and the negative ions. This experiment showing the effect of the particles on the ions from a flame bears out this idea.

And it also seems reasonable that this should be the explanation for the fact that in case of discharge from hot wires the negative ions move more slowly than the positive. In most cases of discharges through gas the negative ions move more rapidly. Some explanation of the behavior here is desirable. If the particles which are driven off have a greater affinity for the negative ions, they will naturally load them to a greater extent than the positive and cause them to move more slowly. As will be shown shortly, we have reason to suppose that the particles of platinum are not driven off to so great an extent in hydrogen as in air and there we find in general that the negative ions move the more rapidly. This fact

also indicates that the presence of the particles is the cause of the slow movement of the negative ions.

The phenomena could also be explained by assuming that the ions are formed from different substances and that the positive ions are given off from some substance which produces more rapidly moving ions. Indeed for some time I supposed that some such explanation was the correct one, but in view of all the facts which have been observed it does not seem as reasonable an explanation as the view given above.

This may throw some light on the fact that the positive ions drawn from the arc move more rapidly than the negative ones, but I hope to publish soon some experiments on the discharge from carbon and the discussion of that point may properly be left until then.

Discharge in Hydrogen.—It has been found that platinum does not decrease in weight when heated in hydrogen.[1] The discharge from a wire surrounded by hydrogen was therefore examined, but it gave decided evidence of the presence of particles which loaded the ions and diminished their velocity. The rates of discharge with different currents through the wire are given in Table XV. The potential difference was 30 volts. The same cylinder and the same glass tube were used in these experiments that have been described in connection with Fig. 5, Part I.

TABLE XV.

Current through Wire.	+ Discharge.	− Discharge.
6.5	$.07 \times 10^{-8}$	
7	.35 "	
7.5	.63 "	1.2×10^{-8}
8	1.05 "	7.1 "
8.5	1.5 "	8.8 "
9	1.5 "	8 "
9.5	1.5 "	3.4 "
10	1.5 "	1.5 "

In attempting to record a higher temperature the wire was melted.

The positive discharge occurs at lower temperatures than the negative as in air. This may possibly be due to a film of oxygen

[1] Wied. Ann., 35, 107.

about the platinum which hinders the negative discharge from passing from the wire, but this does not seem a probable explanation since the wire was heated to nearly white heat in the hydrogen for thirty minutes without producing any change in this respect. However in view of the difficulty which Spiers[1] has shown to exist in getting rid of the oxygen film one can not be positive that thus heating the wire would have the desired effect.

The negative discharge is in general larger than the positive. There was, however, a very great decrease in the amount of discharge when the current through the wire was increased from nine ampères to ten. In fact the negative discharge became as small as the positive. This seemed surprising, and another wire was substituted for the one which had been melted. This second piece required a little more current to bring it to the desired temperature, but it showed exactly the same peculiarities as the first.

The wire was heated for several minutes at as high a temperature as possible, but the discharge with the smaller current continued to be as large as ever and to decrease again when the temperature was raised.

In air the rate of discharge when the wire was first heated was larger than it was after the current had been passing for some time. I endeavored to show that the same was true in hydrogen, but the needle of the galvanometer which I was using for these observations required about fifteen seconds to come to rest, and by that time the discharge had reached a constant value.

Though other experimenters have found no decrease in the weight of platinum when heated in hydrogen, it seemed well to test the matter again, using a current as large as that which produced the smaller rate of discharge. But when the wire was heated for about twenty minutes this piece was also melted. The two parts were, however, weighed and their weight was found to be the same to within .1 mg. as that of the wire before it had been heated. It did not seem profitable to repeat further the work which has already been performed on this point.

But this does not, of course, prove that no particles are driven off, since we know that they are driven off when the wire is in air at

[1] Phil. Mag. (5), 49, 84.

temperatures too low to cause any appreciable loss in weight. The hydrogen about the wire was, therefore, examined for nuclei with water condensation. Moist hydrogen was passed into the tube and then suddenly drawn off and the cloud formation was as noticeable as in air, even though the wire was not heated above red heat. After the condensation had been produced a few times it ceased to be visible until the wire was again heated. Evidently the formation of cloud is a more delicate test of the presence of particles than either the loss in weight or the decrease in the velocity of the ions.

This experiment seems to make it probable that the decrease in the velocity of the ions was caused by particles driven off from the platinum and loading the ions. The particles probably are not driven off to as great an extent as they are in air, since no loss of weight has been detected, and since the velocity of the ions in the hydrogen is much greater than it is in the air.

The fact that the negative discharge in the hydrogen was greatly affected while the positive was not indicates the same thing. If a new kind of ionization was being produced at the higher temperatures, we should expect some indication of its presence in the positive discharge. On the other hand we have already had evidence to show that the particles driven from the platinum have a greater attraction for the negative ions than for the positive, and this would easily explain the phenomena as they here occur.

The particles sent off in hydrogen are no doubt platinum and not an oxide. It would seem highly improbable that a platinum oxide would be formed in the presence of hydrogen. It seems, therefore, probable that the particles sent off in the air are also particles of platinum instead of any oxide of platinum.

Observations were taken with different potential differences and the rate of discharge was found to vary much as it did in air.

This fact seems to show us that the velocities may here be computed as they were in air. When this is done the highest value of K, the velocity of the positive ions, as computed from Table XVIII. was found to be 32.6 cm. per second as compared with 4.5 cm. in air, and that of the negative ions 195 cm. as compared with 3.5. These values seem to be very large, but the data taken certainly indicate such velocities.

Discharge in Oxygen, CO_2 and CO.—With other gases than hydrogen the decrease in the rate of discharge from its first value was very much the same that it was in air. The discharge in oxygen became even smaller than in air enclosed in the same tube. The velocity of the ions was apparently only .2 cm. per second, but no importance can be given to this value, since it depended to so great an extent on the size of the tube and length of time that the wire was heated.

The discharge in CO_2 behaved very much the same as it did in air. In CO the discharge did not decrease as much as in air, but still the behavior of the discharge in this gas was much more similar to its behavior in air than to that in hydrogen. No attempt was made to secure any of these gases chemically pure except hydrogen. In that case no difference was seen between the discharge in hydrogen made from commercial HCl and zinc, and that made from chemically pure HCl and zinc.

Discharge in a Vacuum.—I then proceeded to examine the rate of discharge in a vacuum. It has been shown by others that the rate of discharge from a hot platinum wire increases as the pressure of the surrounding gas is diminished. This was first verified, but in doing so it was found that the discharge at any given pressure of the gas was greatest when the wire was heated for the first time at that particular temperature. In one of the first experiments with the pressure of about 1 mm., with a current of 5 ampères passing through the wire, and a potential difference between the wire and the cylinder of 40 volts the discharge at the beginning was fully 8.4×10^{-7} ampère. This fell in a few minutes to 4.2×10^{-8} ampère, only $\frac{1}{20}$ of its first value. These measurements were taken with a galvanometer whose constant was 4.2×10^{-8} ampère per scale division.

At first sight this would seem to be nothing different from what was found at atmospheric pressure. We there found the rate of discharge to diminish as the wire was heated in a closed tube for some time. The same thing occurs here but to a more marked degree. But further examination seemed to indicate that this phenomenon was essentially different from the other. Allowing the air to enter the tube and then drawing it off did not at all bring the

wire back to its original condition. It no longer gave off as much discharge as it did in the beginning. Allowing the wire to remain over night undisturbed did not restore its power of discharge.

Effect of Hydrogen on the Wire.—Moreover heating the wire in a flame from gasolene restored it partially to its original condition and heating it to a white heat in hydrogen almost entirely restored it. But before any exact measurements were taken with this wire it was accidentally melted and another one was put in its place. The second wire behaved quite differently from the first. The rate of discharge was but little larger when it was first heated in a vacuum from what it was afterwards. Heating it in hydrogen had but little effect upon it. Both moist and dry hydrogen were tried, but neither of them affected the wire. It seemed as if some mistake must have been made in the first observations, but upon trying different pieces of platinum it was found that there was much variety in their behavior.

Platinum was accordingly secured from three different sources. It all contained more or less iridium. Two pieces which were said to contain approximately 1½ per cent. and 5 per cent. of iridium showed a decrease from about 7×10^{-7} ampère to 1.7×10^{-7} ampère when first heated, the potential difference being 50 volts and the current through the wire 5 ampères. The third, containing approximately 10 per cent. of iridium gave a discharge at first of over 8.4×10^{-6} ampère, and then decreased to about the same as the others. All three of these were brought back to nearly their original condition by being heated in hydrogen and they were not affected by being heated in air.

All of the wires were heated in the air at atmospheric pressure for a few minutes before they were heated in a vacuum, so that we would expect any dirt to have been burnt off. But even if one were to assume that the effect was due to some impurity in the wire this alone would not explain the fact that the hydrogen had so great an effect in restoring the wire to its original condition.

A piece of wire having 5 per cent. of impurity, the kind of impurity not being given, was then tried and this was found to have a greater rate of discharge at the beginning than any of the others and a greater subsequent decrease. Heating it in hydrogen produced a

large increase in the rate of discharge, although it lacked much of bringing it back to its original condition.

A piece of wire having 10 per cent. of impurity showed a smaller rate of discharge at first and a smaller change from this than the wire containing only 5 per cent. of impurity.

No attempt was made to go further with this part of the work. The difference in behavior in the different pieces of wire may have been due to a very slight impurity or possibly to gases previously absorbed by the wire. It would probably be a long and tedious piece of investigation to trace out the cause of this behavior and I am unable at present to perform it.

Experiments with Palladium.—Palladium is known to absorb greater quantities of hydrogen than platinum. A few experiments were, therefore, performed with it. But palladium melts at a lower temperature than platinum and on that account no very satisfactory results could be obtained. It melted before any negative discharge was produced and it seemed to be especially liable to melt when heated in hydrogen. As far as was found palladium behaved the same as platinum. The discharge was largest at first, decreased after a short time and was largely restored by heating in hydrogen. The final rate of discharge from it was about the same as that from platinum.

The effect produced by heating the wire in hydrogen throws some light on the mechanism of discharge, but the discussion of this will be deferred until other experiments have been described.

Observations were then made on the rate of discharge in a vacuum under varying conditions. No great exactness can be expected here since the rate of discharge depends to such a large extent upon the length of time that the wire has been heated in a vacuum, but after a few minutes the rate of discharge became reasonably constant and observations could be made of the effects produced by varying the other conditions.

The Current Varied with Wire in Vacuum.—The first quantity varied was the amount of current flowing through the wire. The potential difference was 45 volts and the pressure of the air 1 mm. The other conditions were the same as in the preceding experiments. The results of these observations are given in Table XVI. Column

1 gives the amount of current flowing through the wire, column 2 the positive discharge measured in ampères, column 3 the negative.

TABLE XVI.

Current through Wire.	+ Discharge.	− Discharge.
4.3	$.42 \times 10^{-8}$	$.21 \times 10^{-8}$
4.5	.63 "	1 "
4.8	1.6 "	3.7 "
5	3.3 "	10 "
5.2	6.6 "	22 "

With larger currents the wire was liable to melt. The same current, of course, produces a higher temperature in a vacuum than in air.

The negative current is larger under these conditions. It is also noticeable that the effect of increasing the current is not the same when the wire is in a vacuum that it is in air at ordinary pressures. In air the discharge soon reached a maximum, while here it continually increases.

Potential Varied.—A still greater difference between the phenomena in air and in a vacuum was shown when the potential difference between the wire and the cylinder was varied. Table XVII. gives the rates of discharge with different potential differences. The current was 5.2 ampères and the other conditions were the same as in the preceding experiment. Column 1 gives the potential difference between the wire and the cylinder, column 2 the rate of positive discharge measured in ampères, column 3 that of the negative.

TABLE XVII.

Potential Difference.	+ Discharge.	− Discharge.
45	4.7×10^{-8}	19×10^{-8}
36	3.6 "	15 "
26	3.6 "	10 "
18	3.1 "	8.4 "
9	2.9 "	8.4 "
5	2.5 "	8.4 "
1.5	1.9 "	8.2 "

In the last reading the ground was connected to one end of the wire running into the tube. A shunt could have been placed about

the wire and different points on this grounded, but this would have made one part of the wire positive and the rest negative. No conclusion could have been drawn from observations thus made.

The current here decreased but very little while the potential difference was changed from 45 to 1.5 volts. This was entirely different from the decrease shown when the potential difference was diminished at atmospheric pressure. The discharge at that time decreased very rapidly.

An attempt was made to determine the potential gradient between the hot wire and the surrounding cylinder by means of a second wire. The difficulty to in finding the potential by this means has already been pointed out[1] and in this case the difficulty appeared to be insuperable. All of the space within the tube becomes filled with ions. Moreover the surface of the insulator within the tube ceases to some extent to insulate when the discharge is taking place.

The observations taken seemed to indicate that the potential about the wire approaches the potential of the wire when the pressure of the gas is diminished, but after some study of the leakage to the exploring wire, I decided that it was impossible to make any definite statement concerning it.

Such a change in the potential as that which appeared to take place, could be explained by supposing that all of the space within the tube becomes partially ionized. Even at atmospheric pressures the air becomes ionized to more than molecular distances and the same thing may occur to a much greater extent in a vacuum. But it would be useless to attempt to decide this point, until we have more definite knowledge of the condition of the gas.

The Velocity of Ions in a Vacuum.—The way in which the discharge varies when the potential difference is varied is of itself sufficient to show us that the mechanism of discharge is not the same in a vacuum that it is in air at ordinary pressures. Either the velocity of the ions is so great that their presence does not greatly affect the potential about the wire, or the ionization occurs throughout the gas. In either case the task of finding the velocity of the ions in a vacuum is very difficult. The velocity can not be determined by blowing air past the wire. And until we know whether

[1] PHYS. REV., 12, 67.

the ionization all takes place at the surface of the wire or throughout the gas the employment of an alternating potential difference would give us no definite knowledge.

I had intended to use the method depending upon the determination of the rate of discharge and of the potential gradient about the wire. But evidently we have no right to assume that the potential gradient is the same here that it is in air at higher pressures. Even when the potential difference was only 1.5 volts the rate of discharge did not vary at all as it did at atmospheric pressure.

The water dropper can not be used to determine the potential gradient and the attempt to determine this by a wire has been described.

The method which has been used by J. J. Thomson[1] in finding the velocity of the ions could perhaps be used, but it would require much care. Among other measurements the kinetic energy of the ions carrying the discharge must be determined. As far as can be foreseen this would be much smaller in the case of discharge from a wire than in the case of cathode discharge. The work might possibly be performed, but I have made no attempt whatever to do so.

The Vacuum Varied.—When the attempt was made to secure a series of readings with different pressures of the gas, it was found that no two sets were ever identical. In the first place the work was complicated by the cooling effect of the convection currents about the wire. The same current through it raises it to a higher temperature when there is less gas present. In making observations the temperature should be kept constant. In order to do this a high resistance galvanometer was place in shunt with the wire, and the potential difference at the ends of the wire was measured by the galvanometer. It was assumed that the temperature would be constant, if the resistance were constant, and the resistance is proportional to the ratio of the potential difference measured by the galvanometer and the current through the wire measured by an ammeter. In making the observations this ratio was, accordingly, kept constant.

But even with this precaution I secured widely different sets of readings. At atmospheric pressure the discharge depends largely upon the number of particles about the wire, and with very low

[1] Phil. Mag. (5), 44, 302.

pressures it appears to depend on the amount of occluded hydrogen. These probably were the causes of the wide divergence in the data secured. However, the data which were taken in one case are given in Table XVIII. At each pressure of the gas the wire was heated until the discharge appeared to be constant.

It was found that with lower pressures the particles did not appear to check the motion of the ions. At least the discharge did not then become smaller when the wire was heated for some time. Thus for the particular wire used for securing the data given in Table XVIII. the discharge decreased with time until the pressure was diminished from 50 cm. to 40 cm. After that it continued to be large without regard to the length of time that the wire was heated.

Different pieces of wire behaved differently in this respect as in other respects, some of them apparently sending out the particles at lower pressures than others. In fact it became apparent that it would be useless to attempt to secure exact data on any of these phenomena without a thorough study of the impurities that may exist in the wire.

Column 1 in Table XVIII. gives the pressure of the gas in cm. of mercury. Column 2 the current through the wire, column 3 the positive discharge in ampères, column 4 the negative. The potential difference between the wire and the cylinder was 30 volts. The cylinder about the wire was the same that had been used in preceding experiments.

TABLE XVIII.

Pressure of Gas.	Current through Wire.	+ Discharge.	− Discharge.
72	6.2	$.056 \times 10^{-9}$	$.03 \times 10^{-9}$
50	6.0	.086 "	.05 "
40	5.9	.58 "	.30 "
30	5.8	1.4 "	.84 "
20	5.7	3.1 "	3.7 "
10	5.7	6.3 "	10. "
5	5.6	14. "	25. "
2	5.5	28. "	50. "
1	5.4	50. "	160. "
.5	5.3	160. "	360. "

These data agree with those of others in showing that the negative discharge is greater than the positive when the wire is in a vacuum and heated sufficiently high.

A smaller tube was hermetically sealed and the gas exhausted until cathode rays were produced. The behavior of this tube was but little different from the preceding. The first discharge was very large and continued to decrease for a long time, the final value being little larger than the value it had at 1 mm. pressure. The size of the cylinder also had little to do with the amount of discharge.

The Mechanism of Discharge.—It has been suggested that the discharge is caused by dirt on the wire. This may have some effect on the large discharge produced when the wire is first heated, but it hardly seems possible that it can be the only cause of discharge. Rubbing dirt on the wire before it was heated seemed to produce no effect on the rate of discharge. The different wires did, indeed, behave differently, and yet if the whole effect had been produced by dirt, the phenomena would not have been at all what they were.

A second explanation is one suggested by Berliner[1] that the discharge is produced by occluded hydrogen. If we modify this statement and say that the positive discharge is largely produced in this way, much may be said in favor of the suggestion. At lower temperatures the ionization appears to be produced within or very near to the surface of the metal. This part, at least, of the ionization may be produced by the occluded hydrogen.

This explains the fact that the discharge is so much larger when the wire is first heated in a vacuum. The hydrogen which is driven out from the wire would be largely ionized. Heating the wire in hydrogen would, of course, largely restore to it the power to produce a large discharge, and heating it in air would not.

We would expect the gases within the metal to be ionized at the lower temperatures. At higher temperatures the gas surrounding the wire would begin to be ionized. This would produce the ionization which we found could be blown from the wire. This ionization about the wire would allow the negative discharge to occur, since both positive and negative ions would be produced beyond the surface layer in which the fall of potential is supposed to exist.

The ions from the occluded hydrogen would undoubtedly move more rapidly than those produced in the air. In the case of dis-

[1] Wied. Ann., 33, 289.

charge produced by X-rays the positive ions of the hydogen move more rapidly than the negative of other gases. By supposing that the more rapidly moving positive ions are ions from the occluded hydrogen and that the slower ones are from the air, several other facts could be explained, as, for example, that the average velocity of the ions becomes less at higher temperatures and that the velocity of positive ions in air is greater than the negative, but the action of the particles driven off from the wire would also explain these facts, and it seems more probable that their action is the predominant one.

Possibly the particles which are driven off may ionize the air about the wire and are thus the cause of the slower ions, first, directly, by loading ions already formed, and second, indirectly, by producing a new class of ions.

The fact that a negative discharge in a vacuum is affected by the length of time that the wire is heated shows that the negative discharge is not all produced by the ions about the wire. Apparently some of the discharge is caused by something coming from within the wire.

In all probability neither the positive nor the negative discharge is produced by any one cause. It seems safe to say that the positive ions produced at the lower temperature come from within the wire, and that some at least of the negative ions are produced in the gas about the wire. It is also safe to say that some of the ions that come from within the wire are due to occluded hydrogen, and that particles driven off from the wire produce some of the phenomena. Probably the different causes enter in different amounts with different pieces of wire.

Summary.—The particles producing the decrease in the velocity of the ions come from the hot platinum and not from the copper connections.

The particles from a hot platinum wire have a greater retarding action on the negative ions drawn from a flame than on the positive.

In hydrogen the rate of discharge is much larger than in air. The positive discharge still begins at lower temperatures than the negative. The negative soon surpasses the positive, passes through a maximum and rapidly decreases, coming to have about the same value as the positive when very near the melting point. The

presence of particles about the wire after it had been heated was also shown by the cloud formation when the hydrogen was suddenly cooled, but no loss in the weight of the wire was detected. Apparently particles are driven off, but to a much smaller extent than in air. The largest value for K_1 was found to be 42.6 cm. per sec. for a potential gradient of 1 volt per cm., as compared with 4.5 cm. in air and for K_2 195 cm. as compared with 3.6 cm. As in air the rates of discharge increased very rapidly as the potential differences were increased.

The behavior of the wire in oxygen, CO_2 and CO is practically the same as in air.

The experiments seemed to indicate that the particles are driven off as platinum and not as some oxide of platinum.

When a platinum wire is first heated in a vacuum the rate of discharge is much greater than at any time afterwards. Heating the wire in hydrogen partially restored to it the power of producing a large discharge. But this did not produce any effect on the negative discharge.

The rate of discharge is much greater in a vacuum than in air at atmospheric pressures.

The positive discharge at higher temperatures is less in a vacuum than the negative. The amount of both positive and negative discharges continually increase as the temperature of the wire is increased.

The potential difference between the wire and cylinder about it has little effect on the amount of discharge in a vacuum. In this respect it behaves entirely different from the wire in air at atmospheric pressures. The size of the surrounding cylinder has little effect on the amount of discharge. No method was found by which the velocity of the ions in a vacuum could be determined.

The phenomena indicate that the ionization first occurs within the wire, and there are some reasons for thinking that this part of the ionization is produced from occluded hydrogen.

CONTRIBUTIONS TO THE STUDY OF THE INDUCTION COIL.

BY JAMES EDMUND IVES

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§ 1. INTRODUCTION.

THE induction coil has been investigated in recent years by Colley, Oberbeck, Walter, Johnson and others, both theoretically and experimentally. A list of the more important recent contributions to the subject is given at the end of this paper.

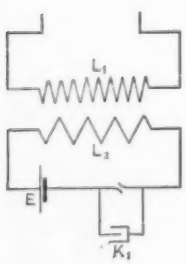

Fig. 1.

The induction coil is an electromagnetic system with two degrees of freedom. It is usually constructed by winding a *secondary* coil of a large number of turns of fine wire upon a primary of a few turns of heavy wire. (See Fig. 1.) The core of the primary may be filled with iron wires to increase its inductance. The terminals of the primary are so arranged that they can be conveniently attached to some source of electromotive force, and the primary circuit is provided with a make-and-break apparatus. A condenser of suitable capacity is put around the break, so as to diminish as far as possible the sparking at the break.

The usual method of operating the coil is as follows. A battery having been placed in the primary circuit, the circuit is opened and closed periodically. The condenser around the break is then ad-

justed so that the sparking at the break is reduced to a minimum. The secondary circuit can be left open, or can be closed through a vacuum tube, an electrolytic cell, or in any other way that is desired.

The action of the coil may be briefly stated. When the primary circuit is closed, the current begins to grow, and soon reaches what is practically its maximum value. This *make current* in the primary induces an electromotive force in the secondary, which, however is not of any importance. The primary circuit is now broken, and owing to its capacity, that is, to the capacity around the break, dies away in a series of oscillations, whose period is constant, but whose amplitude is rapidly decreasing. In a small induction coil they are so rapid that there are 1,000 to 10,000 of them in a second. These oscillations of the primary current set up an induced electromotive force in the secondary, which depends for its magnitude, as will be shown later, upon the constants of the coil, the initial strength of the primary current, the velocity of breaking the primary, and the capacity of the condenser around the break.

The usual method of using a coil is with a slow break, the primary being broken less than a hundred times a second. In this case the current in the primary rises to approximately its full value before the break occurs, and the oscillations due to the break die away before the next make. The current under these conditions would be represented by Fig. 2. This case will evidently be completely covered if a *single* break is considered.

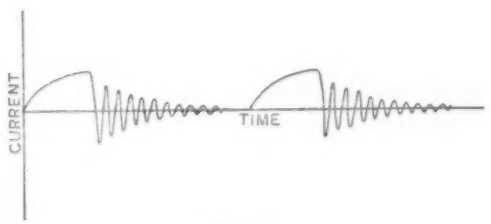

Fig. 2.

If the break is *not* slow, as in the case of the Wehnelt interruptor, the theory becomes more complicated, as the make current does not have time to rise to a steady value before the break occurs, and the break current also overlaps the next make current.

The two simplest cases will be treated in this paper, viz.: first, the case of a single break, and second, the case of a simple harmonic electromotive force in the primary.

§ 2. THEORY OF THE COIL.

In general both the primary and secondary circuits have inductance, resistance and capacity.

It will be seen from Fig. 1, that when the primary circuit is closed, its capacity may be considered infinite, but when it is broken, its capacity is that of the condenser K.

The capacity of the secondary circuit in the ordinary mode of using the induction coil is the distributed capacity due to the juxtaposition of successive turns of the coil. This, although very small, is not infinitesimal. This question of the distributed capacity of the secondary will be discussed more fully further on in this paper.

Theory of One Circuit for a Single Break.

Since the theory of a *single* circuit containing an inductance, resistance, electromotive force, and a condenser around the break is closely related to that of the primary of an induction coil, a brief discussion of the single circuit will be given before the discussion of two circuits. The same conditions will be supposed to hold as for the primary, viz: the breaks and makes will be sufficiently far apart so that the oscillations due to the break have died away before the make occurs.

We will then suppose the current to have risen to its maximum value before the break occurs, and will consider what takes place after breaking it.

The well-known equation for such a circuit is the following:

$$L\frac{dI}{dt} + RI + \frac{1}{K}\int Idt = E, \tag{1}$$

where I is the current; t, the time; L, the inductance; R, the resistance; K, the capacity around the break; and E the steady impressed electromotive force.

This equation may be written in the form

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{K} = E, \tag{2}$$

where q is the quantity of electricity which has passed any cross-section of the circuit during the time t or, what is the same thing, the quantity of electricity which has entered the condenser in the time t. If we bring E over to the left hand side of the equation, and write

$$q' = q - Q$$

where $Q = KE$ is the steady charge of the condenser under the electromotive force E, equation (2) becomes

$$L\frac{d^2q'}{dt^2} + R\frac{dq'}{dt} + \frac{q'}{K} = 0. \tag{3}$$

This is the well-known equation for a system with one degree of freedom.[1]

It is found that the charge is oscillatory or non-oscillatory according as

$$R^2 < 4\frac{L}{K} \quad \text{or} \quad R^2 > 4\frac{L}{K}.$$

In this discussion we shall consider only oscillatory discharges, since the induced electromotive force in the secondary of an induction coil due to a non-oscillatory discharge is so small that it is of no practical importance.

The solution for (3) is given by

$$q' = e^{\mu t}(A\cos \nu t + B\sin \nu t) \tag{4}$$

where

$$\mu = -\frac{R}{2L},$$

$$\nu = \sqrt{\frac{1}{KL} - \frac{R^2}{4L^2}}.$$

The arbitrary constants A and B will be determined by the initial conditions, which in the case we are considering are the following:

When $t = 0$, $q' = -Q$ and $I = I_0$.

[1] See Webster, Elect. Mag., pp. 484–488.

Whence it follows that

$$A = -Q$$

$$B = \frac{I_0 + \mu Q}{\nu}.$$

Therefore since $q' = q - Q$ we have

$$(5) \qquad q = Q - e^{\mu t}\left[Q \cos \nu t - \frac{I_0 + \mu Q}{\nu} \sin \nu t\right],$$

which may be written

$$(6) \qquad q = Q - e^{\mu t}\left[Q^2 + \left(\frac{I_0 + \mu Q}{\nu}\right)^2\right]^{\frac{1}{2}} \cos (\nu t + \alpha)$$

where

$$\alpha = \tan^{-1}\frac{I_0 + \mu Q}{\nu Q}.$$

For the current we have

$$(7) \quad I = \frac{dq}{dt} = e^{\mu t}\left[Q^2 + \left(\frac{I_0 + \mu Q}{\nu}\right)^2\right]^{\frac{1}{2}} [\nu \sin (\nu t + \alpha) - \mu \cos (\nu t + \alpha)]$$

and for the difference of potential between the two sides of the condenser

$$(8) \qquad V = \frac{q}{K} = E - \frac{e^{\mu t}}{K}\left[Q^2 + \left(\frac{I_0 + \mu Q}{\nu}\right)^2\right]^{\frac{1}{2}} \cos (\nu t + \alpha).$$

In the case which is usually considered, $E = 0$, and equations (6), (7), and (8) become

$$(9) \qquad q = \frac{I_0 e^{\mu t}}{\nu} \sin \nu t$$

$$(10) \qquad I = \frac{I_0 e^{\mu t}}{\nu} [\nu \cos \nu t + \mu \sin \nu t]$$

$$(11) \qquad V = \frac{I_0 e^{\mu t}}{K\nu} \sin \nu t.$$

Theory of one Circuit. Harmonic E.M.F.

If instead of the circuit being broken, we have impressed upon it a periodic electromotive force $E_0 \cos \omega t$ we shall have instead of equation (2)

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{K} = E_0 \cos \omega t. \tag{12}$$

The solution of which is given by

$$q = \frac{E_0}{\omega\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \cos(\omega t - \alpha), \tag{13}$$

$$I = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \sin(\omega t - \alpha), \tag{14}$$

$$V = \frac{E_0}{\omega K\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}} \cos(\omega t - \alpha). \tag{15}$$

V is the difference of potential between the two sides of the condenser. V_m the maximum difference of potential is given by

$$V_m = \frac{E_0}{\omega} \frac{1}{K\sqrt{R^2 + \left(L\omega - \frac{1}{K\omega}\right)^2}}. \tag{16}$$

Theory of two Circuits for a Single Break.

The theory of the induction coil is the theory of two circuits. The well-known equations for two circuits, are[1]

$$\begin{aligned} L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt} + R_1I_1 + \frac{1}{K_1}\int_0^t I_1 dt &= E_1, \\ L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt} + R_2I_2 + \frac{1}{K_2}\int_0^t I_2 dt &= E_2. \end{aligned} \tag{17}$$

In the case of the induction coil, since there is no impressed electromotive force in the secondary, $E_2 = 0$. Also since the impressed electromotive force in the primary is usually small com-

[1] See Webster, Elect. Mag., pp. 491–502.

pared with the induced electromotive force in the primary, E_1 may be neglected. Then equations (17) may be written

$$(18)\qquad \begin{aligned} L_1\frac{d^2q_1}{dt^2} + M\frac{d^2q_2}{dt^2} + R_1\frac{dq_1}{dt} + \frac{q_1}{K_1} &= 0, \\ L_2\frac{d^2q_2}{dt^2} + M\frac{d^2q_1}{dt^2} + R_2\frac{dq_2}{dt} + \frac{q_2}{K_2} &= 0. \end{aligned}$$

Equations (18) are the equations for a system with two degrees of freedom, and no impressed forces.

It is well known from the theory of such equations[1] that the displacements q_1 and q_2, which are here quantities of electricity, may either die away without oscillations, or may die away with a series of oscillations. The first will happen if R_1 and R_2 are both very large. The second case is the case of the induction coil, and the one in which we are interested.

The solutions of these equations may be written down immediately, in the following form:

$$(19)\qquad \begin{aligned} q_1 &= e^{-\alpha t}(A_1\cos\beta t + B_1\sin\beta t) + e^{-\gamma t}(C_1\cos\delta t + D_1\sin\delta t), \\ q_2 &= e^{-\alpha t}(A_2\cos\beta t + B_2\sin\beta t) + e^{-\gamma t}(C_2\cos\delta t + D_2\sin\delta t). \end{aligned}$$

In which α, β, γ and δ are given by

$$(20)\qquad \begin{aligned} \alpha &= \frac{R_1L_2 + R_2L_1}{4m} + \frac{R_1[L_2n - 2K_1m] + R_2[L_1n - 2K_2m]}{4m\sqrt{n^2 + LM^2K_1K_2}}, \\ \gamma &= \frac{R_1L_2 + R_2L_1}{4m} - \frac{R_1[L_2n - 2K_1m] + R_2[L_1n - 2K_2m]}{4m\sqrt{n^2 + LM^2K_1K_2}}, \\ \alpha^2 + \beta^2 &= \frac{n + \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}{2K_1K_2m}, \\ \gamma^2 + \delta^2 &= \frac{n - \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}{2K_1K_2m}. \end{aligned}$$

Where

$$m = L_1L_2 - M^2,$$

$$n = L_1K_1 + L_2K_2.$$

[1] Webster, loc. cit., pp. 494–498.

In finding the values of α, β, γ, δ we have neglected the product $\alpha\gamma$ as small compared with the product $\beta\delta$. This is justified since α and γ are the reciprocals of the time constants of the two circuits and β and δ are the reciprocals of the periods multiplied by 2π, and the periods are small compared with the time constants.

Since equations (19) can be written

$$(21)\qquad \begin{aligned} q_1 &= e^{-\alpha t}A\cos(\beta t - \theta_1) + e^{-\gamma t}B\cos(\delta t - \theta_2),\\ q_2 &= e^{-\alpha t}C\cos(\beta t - \theta_3) + e^{-\gamma t}D\cos(\delta t - \theta_4), \end{aligned}$$

where

$$A = \sqrt{A_1^2 + B_1^2},\quad B = \sqrt{C_1^2 + D_1^2},\quad C = \sqrt{A_2^2 + B_2^2},\quad D = \sqrt{C_2^2 + D_2^2},$$

and

$$\theta_1 = \tan^{-1}\frac{B_1}{A_1},\quad \theta_2 = \tan^{-1}\frac{D_1}{C_1},\quad \theta_3 = \tan^{-1}\frac{B_2}{A_2},\quad \theta_4 = \tan^{-1}\frac{D_2}{C_2}.$$

we see that q_1 and q_2 and therefore the currents, are each sums of two damped harmonic oscillations. The coefficients, A, B, C, D, give the amplitudes of the component oscillations in each circuit.

The constants of equations (19) and (21) must be determined from the initial conditions, viz: when

$$t = 0,\quad q_1 = 0,\quad I_1 = \frac{dq_1}{dt} = I_0,$$

$$q_2 = 0,\quad I_2 = \frac{dq_2}{dt} = 0.$$

Boynton[1] has determined these constants for the case of the Tesla coil, and Colley, Oberbeck, Walter and Johnson have found approximate values for the case of the induction coil.

In the induction coil as ordinarily used there is no condenser in the secondary, and the only capacity it has is the distributed capacity due to the juxtaposition of the successive turns of the coil. This has been found to be very small in ordinary coils. If a coil is wound bifilarly the capacity is very much increased, and the inductance decreased.

[1] PHYS. REV., Vol. VII., pp. 35–48, 1898.

Max Wien[1] found for three coils that he examined the following results:

Coil No. 1. $R = 2921$ siemens,

$L = 3.97$ henries,

Distributed capacity = .00077 microfarads.

Coil No. 2. $R = 69$ siemens,

$L = .199$ henries,

Distributed capacity = .0019 microfarads.

Coil No. 3 (wound bifilarly).

$R = 135.1$ siemens,

$L = .767$ henries,

Distributed capacity = .0139 microfarads.

Walter[2] found for the secondary of an induction coil having an inductance of about 500 henries a capacity of only about 1.1 millionths of a microfarad; and for the secondary of a 60 cm. induction coil, having an induction of about 9,000 henries, a capacity of about 6.5 millionths of a microfarad.

Oberbeck[3] assumes that the capacity of the secondary of a small induction coil is that of a small Leyden jar, *i. e.*, about .0005 of a microfarad.

The theory of the distributed capacity of a coil has not yet been worked out, but it seems probable that the capacity is very small. The experiments which will be described later seem to indicate this.

Approximate expressions for the difference of potential in the secondary, and for the primary current have been obtained by neglecting small quantities.

Walter[4] and Oberbeck[5] have obtained the following expressions for the case of the open secondary.

[1] Wied. Ann., Bd. 44, p. 712, 1891.
[2] Wied. Ann., Bd. 66, pp. 628–630, 1898.
[3] Wied. Ann., Bd. 64, p. 204, 1898.
[4] Wied. Ann., Bd. 62, pp. 307–322; Bd. 66, pp. 623–635.
[5] Wied. Ann., loc. cit.

For the difference of potential between the terminals of the secondary they get

$$(22) \qquad V_2 = \frac{I_0 M}{L_2K_2 - L_1K_1}\left[\sqrt{L_2K_2}\, e^{-\frac{R_2}{2L_2}t} \sin \frac{t}{\sqrt{L_2K_2}} - \sqrt{L_1K_1}\, e^{-\frac{R_1}{2L_1}t} \sin \frac{t}{\sqrt{L_1K_1}}\right].$$

The periods of the component oscillations are evidently given by

$$(23) \qquad T_1 = \pi\sqrt{L_1K_1},\ T_2 = \pi\sqrt{L_2K_2}.$$

If we consider only the first few oscillations, the damping may be neglected and (22) becomes

$$(24) \quad V_2 = \frac{I_0 M}{L_2K_2 - L_1K_1}\left[\sqrt{L_2K_2} \sin \frac{t}{\sqrt{L_2K_2}} - \sqrt{L_1K_1} \sin \frac{t}{\sqrt{L_1K_1}}\right].$$

Three important cases may be distinguished according as L_1K_1 is large compared with L_2K_2; is equal to L_2K_2, or is small compared with L_2K_2.

In the first case (24) reduces to

$$(25) \qquad V_2 = \frac{I_0 M}{\sqrt{L_1K_1}} \sin \frac{t}{\sqrt{L_1K_1}}.$$

In the second, to

$$(26) \qquad V_2 = \frac{I_0 M}{2\sqrt{L_1K_1}} \sin \frac{t}{\sqrt{L_1K_1}} = \frac{I_0 M}{2\sqrt{L_2K_2}} \sin \frac{t}{\sqrt{L_2K_2}}.$$

In the third, to

$$(27) \qquad V_2 = \frac{I_0 M}{\sqrt{L_2K_2}} \sin \frac{t}{\sqrt{L_2K_2}}.$$

Assuming that there is no magnetic leakage,

$$M = \sqrt{L_1L_2}$$

and we get for the maximum difference of potential in the three cases

$$25a \qquad V_2 \max = I_0 \sqrt{\frac{L_2}{K_1}},$$

$$26a \qquad V_2 \max = \frac{I_0}{2} \sqrt{\frac{L_2}{K_1}} = \frac{I_0}{2} \sqrt{\frac{L_1}{K_2}},$$

$$27a \qquad V_2 \max = I_0 \sqrt{\frac{L_1}{K_2}}.$$

Formula $25a$ assumes that the capacity of the secondary can be neglected. Walter[1] found by experiment that this was true for small coils. The larger the coil the more will the potential deviate from this value and approach those given by $26a$ and $27a$.

For the primary current, neglecting the reaction of the secondary, Walter[2] obtains

$$(28) \qquad I_1 = I_0 e^{-\frac{R}{2L_1}} \cos \frac{t}{\sqrt{L_1 K_1}}.$$

The period of the primary current is therefore given by

$$(29) \qquad T_1 = 2\pi \sqrt{L_1 K_1}.$$

For the maximum difference of potential between the plates of the primary condenser, during the oscillations of the primary current, Walter[3] gives the following expression:

$$(30) \qquad V_{1\,\max} = I_0 \sqrt{\frac{L_1}{K_1}}.$$

Equation (30) shows that if the capacity of the primary circuit is very small, the induced electromotive force in this circuit is very great.

[1] Loc. cit.
[2] Wied. Ann., Bd. 62, p. 310, 1897.
[3] Loc. cit., p. 322.

Johnson[1] makes a correction for the reaction of the secondary and writes (29) and (30) as follows:

29*a* $$T_1 = 2\pi \sqrt{L_1 K_1 \left(1 - \frac{M^2}{L_1 L_2}\right)}$$

and

30*a* $$V_{1\,\max} = I_0 \sqrt{\frac{L_1}{K_1}\left(1 - \frac{M^2}{L_1 L_2}\right)}.$$

Theory of two Circuits. Harmonic E.M.F.

If, instead of a constant current, which is suddenly broken, we have an alternating current in the primary, equations (17) become

(31)
$$L_1 \frac{d^2 q_1}{dt^2} + M \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{K_1} = E \cos \omega t,$$
$$L_2 \frac{d^2 q_2}{dt^2} + M \frac{d^2 q_1}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{q_2}{K_2} = 0.$$

We will only consider the case in which the primary circuit is a closed circuit containing no condenser, and in which therefore K_1 can be considered infinite.

We obtain for the secondary circuit

(32) $$q_2 = \frac{M\omega^2 E}{\sqrt{D_1^2 + D_2^2}} \cos(\omega t - \alpha)$$

where

$$D_1 = (L_1 L_2 - M^2)\omega^4 - \left(R_1 R_2 + \frac{L_1}{K_2}\right)\omega^2,$$

$$D_2 = (R_2 L_1 + R_1 L_2)\omega^3 - \frac{R_1}{K_2}\omega$$

and

$$\alpha = \cot^{-1} \frac{D_1}{D_2}.$$

To get V_2 the difference of potential of the condenser in the secondary we write

$$V_2 = \frac{q_2}{K_2}.$$

[1] Wied. Ann., Bd. 3, pp. 450 and 458, 1900.

This gives for V_m, the amplitude of the potential oscillation,

$$V_m =$$

$$(33) \qquad \frac{M\omega^2 E}{K_2\sqrt{\left[(L_1L_2 - M^2)\omega^4 - \left(R_1R_2 + \frac{L_1}{K_2}\right)\omega^2\right]^2 + \left[(R_2L_1 + R_1L_2)\omega^3 - \frac{R_1}{K_2}\omega\right]^2}}$$

In the induction coil, as ordinarily used, with an open secondary, $K_2 = 0$. In this case (33) becomes

$$(34) \qquad V_m = \frac{M\omega E}{\sqrt{L_1^2\omega^2 + R_1^2}}.$$

To find what capacity in the secondary will give the maximum amplitude for the potential oscillation, we must differentiate the denominator of (33) with respect to K_2 and put the result equal to zero. Calling this value of the capacity K_m, we get

$$(35) \quad K_m = \frac{L_1(L_1L_2 - M^2)\omega^2 + R_1^2L_2}{(L_1L_2 - M^2)\omega^4 + (R_1^2L_2^2 + 2R_1R_2M^2 + R_2^2L_1^2)\omega^2 + R_1^2R_2^2}.$$

Experiments verifying this formula will be described in § 6.

§ 3. Instruments and Methods.

The induction coil used in the experiments described in the following pages, was made in separate pieces so that each part of it was independent of every other part, and the number of sections in the secondary could be varied at will.

The primary was made of two layers of heavy double-cotton-covered copper wire, No. 12, Brown & Sharpe gauge, and had the following dimensions:

Number of turns in outer layer,	109½
Number of turns in inner layer,	109
Total number of turns,	218½
Mean length,	25.63 cm.
Diameter of the outer coil,	4.47 cm.
Diameter of the inner coil,	4.11 cm.
Diameter of the wire,	.230 cm.

The primary was made by winding the wire upon a wooden cylinder, about which had been previously wrapped several layers

of heavy paper. To the ends of the paper tube were glued two wooden end pieces to keep the coil from spreading. When the coil had been wound, and the ends of the wire attached to one of the end pieces, the wooden cylinder was withdrawn. The inductance of the primary could be changed by putting iron wires in the hollow core. The iron wires used were of soft iron 35 cm. long and .0830 cm. in diameter. Most of the experiments were conducted without any iron in the core. The self inductance of the primary with an air core was found to be .000281 of a henry. Its resistance at 21° Centigrade, was .1666 of an ohm.

TABLE I.

Mark.	No. of Turns.	Inside Diameter in Centimeters.	Outside Diameter in Centimeters.	No. of Turns in a Layer.	No. of Layers.	Self-Inductance in Henries.	Mutual Inductanc with Primary in Henries.	Resistance at 21° C. in Ohms.
AI	5469	7.62	13.00	52-60	100	3.551		1527.0
AII	4600	7.62	13.00	50	92	2.594		1351.7
BI	5558	7.36	12.85	52-59	100	3.595	.00763	1516.0
BII	5000	7.36	12.49	50-57	95	2.913		1365.0
BIII	5000	7.37	12.82	50	100	2.951		1391.3
BIV	5000	7.34	12.25	51-60	90	2.934		1425.1
CI	1250	7.39	8.88	50	25	.1867		297.8
CII	"	7.36	8.85	"	"	.1896		298.5
CIII	"	7.37	8.87	"	"	.1880		297.5
CIV	"	7.40	8.95	"	"	.1893		295.8
DI	2500	7.38	10.28	50	50	.684		612.2

The secondary was made in eleven sections. Each section was independent of all the others and could be slipped at will on and off a hard rubber sleeve, which itself was slipped over the primary. The sections were all wound with No. 34 (Brown & Sharpe gauge) single-silk-covered copper wire. The diameter of the wire was .01601 cm. The sections were all of the same width, viz: 1.27 cm. (half an inch). Six of them contained about 5,000 turns each; one contained about 2,500 turns and the other four contained about 1,250 turns each. Their dimensions are given in Table I. Table I*a* contains the inductances, self and mutual, of the coils joined up in series.

TABLE I*a*.

Coils Used.	Self Inductance in Henries.	Mutual Inductance with Primary in Henries.
BII + BIII.	9.67	
BII + BIII + BIV.	18.92	
BI + BIV.	11.06	.0145
BI + BIV + BII.	20.84	.0213
CI + CII.	.5774	
CI + CII + CIII.	1.076	
CI + CII + CIII + CIV.	1.639	

All the sections were wound on a form in a lathe, a strip of Banker's Bond paper being placed between each two layers. AI and AII were wound on wooden forms, on which they were permanently retained. All the others were wound on a metal form. After the section had been wound the metal form was removed, and the section boiled in paraffin in a vacuum to exclude all moisture and air. Connections were made by soldering heavier wires to the ends of the sections, and inserting the ends in mercury cups.

The condensers used in these experiments had a total capacity of about thirteen microfarads. The smaller condensers were very carefully put together and were made of alternate sheets of mica and tinfoil. After the condenser had been assembled it was clamped by a metal clamp and boiled in beeswax. The metal clamp was retained permanently. The smaller condensers were divided into ten sections ranging from .006 to 0.15 of a microfarad. A set of 16 sections of a mica and tinfoil condenser, kindly loaned by Messrs. Morris E. Leeds & Co., of Philadelphia, was also used. Each section had a capacity of about .2 microfarad. These were boiled in paraffin and held together by a wooden clamp. Three large Stanley, paper and tinfoil, condensers, each having a capacity of about 2.5 microfarads, were also used.

The capacities of the condensers were determined by making an absolute determination of one section by the tuning fork charge and discharge method[1] and then comparing the other sections with this section by the bridge method.

[1] Thomson & Searle, Phil. Trans., 1890 Vol. A, pp. 583–621.

The *measurements of difference of potential* were all made with a modified Kelvin quadrant electrometer, used idiostatically. The deflections were therefore proportional to the squares of the potential.

The instrument used was the one used by Professor Webster, and described by him in his paper on electrical oscillations.[1] The quadrants were made of brass and the needle of aluminum. The needle was suspended by a quartz fiber and connected to the binding post by a platinum wire dipping into sulphuric acid. The instrument worked excellently as long as the acid was fresh. After the acid had been standing for some weeks it was found to be necessary to boil it for several hours, for if not fresh or recently boiled there was a lag when a reading was taken.

The differences of potential measured were usually small, less than 50 volts, since the primary was nearly always used with an air core. The electrometer as adjusted for these experiments was too sensitive to measure directly potentials greater than 50 volts. It could be arranged however to measure any potential by the introduction of an auxiliary air condenser of adjustable capacity in series with it in the electrometer circuit. In Fig. 3 *A* is the auxiliary air condenser, and *V* the electrometer.

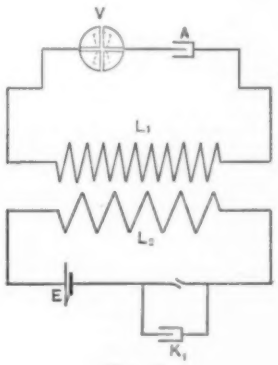

Fig. 3.

The true difference of potential can be calculated from the apparent difference given by the electrometer reading, and the known capacity of the auxiliary condenser.

In all the experiments the electrometer was used idiostatically, one side being connected to earth so as to give a constant zero.

The usual method of using the electrometer was as follows. After taking a series of observations, the instrument was calibrated by noting the deflections due to different known differences of potential. The known differences of potential used were those of a storage battery, determined by a Weston voltmeter. A calibration curve

[1] PHYS. REV., Vol. VI., p. 301, 1898.

was plotted, and the value of the potential corresponding to a given deflection read off directly from the curve.

A curious source of error observed and studied by Hallwachs[1] caused the writer some trouble in the earlier experiments. Since in the idiostatic method of using the electrometer the deflection is proportional to the square of the potential, when the sign of the potential is changed without changing its absolute value the deflection should be unchanged. It was found that this was not quite true, and investigation showed that it was due to the contact difference of potential between the brass quadrants and the aluminum needle, which increased the deflection in one case and decreased it in the other. An approximately correct value for the difference of potential is found by taking the mean between the two deflections. This was done in calibrating the instrument. Hallwachs has shown that an electrometer in which the quadrants and the needle are made of different metals can be used to find the contact difference of potential between the two metals. The difference of potential between the brass quadrants and the aluminum needle was determined in this way by the writer, and found to be equal to .42 of a volt.

In these experiments the electrometer was used idiostatically in two ways, first, with a steady deflection with an alternating current, and second, ballistically with a transient current. In both cases the deflection was proportional to the mean square of the potential. In both cases the potential performed a simple harmonic oscillation, the difference being that in the first case the oscillation was undamped, and in the second case it was strongly damped, the time constant being about a thousandth of a second.

The *measurements of inductance* were made by comparison with a standard coil, the inductance of which had been previously calculated by Maxwell's formula[2] for the self-inductance of a circular coil of rectangular cross-section.

If (see Fig. 4)

n = number of turns in coil.

a = mean radius of coil.

r = diagonal of the cross-section.

θ = angle made by diagonal with lower edge of the cross-section.

[1] Wied. Ann., Bd. XXIX., pp. 1–47, 1886.

[2] See Rayleigh, Phil. Trans., 1882, pt. II., p. 675.

We have, using the first two terms of the series

$$L = 4\pi an^2 \left[\log_e \frac{8a}{r} + \tfrac{1}{12} - \tfrac{4}{3}\left(\theta - \frac{\pi}{4}\right) \cot 2\theta - \tfrac{1}{3}\pi \operatorname{cosec} 2\theta \right.$$
$$\left. - \tfrac{1}{6} \cot^2 \theta \log_e \cos \theta - \tfrac{1}{6} \tan^2 \theta \log_e \sin \theta \right]$$
$$+ \frac{\pi n^2 r^2}{24a} \left[\log_e \frac{8a}{r} (2 \sin^2 \theta + 1) + 3.45 + 27.475 \cos^2 \theta \right.$$
$$\left. - 3.2 \left(\frac{\pi}{2} - \theta\right) \frac{\sin^3 \theta}{\cos \theta} + \tfrac{1}{5} \frac{\cos^4 \theta}{\sin^2 \theta} \log_e \cos \theta + \tfrac{13}{8} \frac{\sin^4 \theta}{\cos^2 \theta} \log_e \sin \theta \right]$$

In calculating the value of L for the standard coil it was found that neglecting the second term would introduce a minus error of .4 of one per cent.

Fig. 4.

The coils used as the standard were two coils of the same dimensions, used by Taylor,[1] in his work on standard cells, as the fixed coils of an electrodynamometer. They are fully described by him in the paper referred to.

The mean value of the self inductance of each coil by calculation from the formula was found to be .001881 of a henry.

The comparison of the inductances to be measured with the standard was made by Maxwell's bridge method using the alternating current and a telephone[2] instead of a galvanometer. The comparison was usually made by comparing the unknown inductance first with the upper coil and then with the lower, and taking the mean of the two values obtained. The two values so obtained usually agreed to about a tenth of one per cent. It was found that with the alternating current used, the inductances could be measured to about a tenth of one per cent. The self inductances of the secondary segments, both separately and combined together are given in Tables I. and Ia.

The mutual inductances of the secondary segments with the primary were measured by Maxwell's[3] bridge method for comparing

[1] PHYS. REV., Vol. VII., pp. 156–159, 1898.
[2] See Max Wien, Wied. Ann., Bd. 44, pp. 681–688, 1891.
[3] See Gray, Absolute Measurements, Vol. II., pp. 465–469.

the mutual inductance of two coils with the known self inductance of one of the coils. In this case also, the alternating current and the telephone were used instead of a steady current and a galvanometer. The telephone method is much more rapid than the galvanometer method. The results are given in Table I. The inductances were all measured without any iron in the core, that is with an air core.

The *measurements of resistance* were made by the usual zero method of the Wheatstone bridge with a steady current and a galvanometer. The resistance box used was one made by Elliott Bros., of London. The resistances of the box were examined by Mr. Frank K. Bailey, of the university, and found to be accurate to a tenth of one per cent. The resistances are given in Table I. reduced to 21° centigrade.

The *measurements of currents* were made for direct currents with a Weston ammeter, and for alternating currents with a hot wire ammeter, which was calibrated by comparison with the Weston ammeter. The currents were measured to about one per cent.

To conduct the experiments described in § 6, it was necessary to obtain a harmonic current that should be free from overtones. This was done by Pupin's[1] method, which consists in inserting inductance and capacity in series in the circuit, and then adjusting the relative amounts of inductance and capacity until the circuit is in resonance for the particular frequency which is desired. When this is so, if the resistance of the circuit is relatively small, the frequency which is desired will so far predominate that the other frequencies cannot be detected. In order to use this method it is necessary to have either a large inductance or a large capacity. Since it was easier to construct a large inductance than a large capacity, a coil was built which gave the required inductance. The dimensions of a coil which would give the desired inductance were calculated from Maxwell's formula on p. 297. The cross-section of the whole coil, drawn to scale, is shown in Fig. 5.

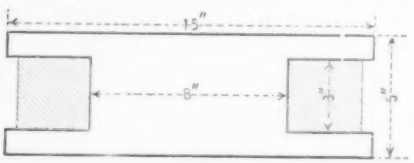

Fig. 5.

[1] Am. Jour. Sci., Vol. XLVIII., pp. 379–389, 473–485, 1894.

The portion occupied by the wire is shaded. This shaded portion was intended to be 3 inches on a side. In the coil as constructed it was 3 inches from top to bottom, and 2¾ inches from side to side. The coil was wound with about two miles of No. 18 (Brown & Sharpe gauge) double cotton insulated copper wire. Diameter, .0403 of an inch. The wire was wound in 60 layers of 60 turns each, making 3,600 turns in all. At the beginning and end of every ten layers, the wire was led off to a binding post on the top of the coil, so that the coil was divided into six independent sections.

The dimensions of the wooden form on which the coil was wound are those shown in Fig. 9, viz:

Outside diameter,	15	inches.
Inside diameter,	8	"
Height of groove,	3	"
Depth of groove,	3½	"
Height of whole coil,	5	"

The dimensions of the coil proper were

Outside diameter,	13½	inches
Inside diameter,	8	"
Height of cross-section,	3	"
Depth of cross-section,	2¾	"

The coil was designed to give the maximum inductance for the length of wire used.[1] Its inductance calculated before construction was 3.33 henries. After construction its inductance was found by comparison with the standard inductance to be 3.01 henries. This is probably partly due to the fact that the designed geometrical conditions, viz.: a rectangular cross-section was not quite fulfilled.

§ 4. The Iron Core.

To determine the effect of increasing the amount of iron in the core, the number of iron wires inserted in the core was varied, the current for any given experiment being kept constant.

An alternating current of about 60 complete periods per second was used in the primary. The difference of potential between the terminals of the secondary was measured with the electrometer, used idiostatically.

[1] See Maxwell, Elect. Mag., Vol. II., p. 316.

Tables II.–V. and Figs. 6–9 show the results obtained. In the curves the differences of potential in the secondary are plotted as ordinates, and the number of wires as abscissas.

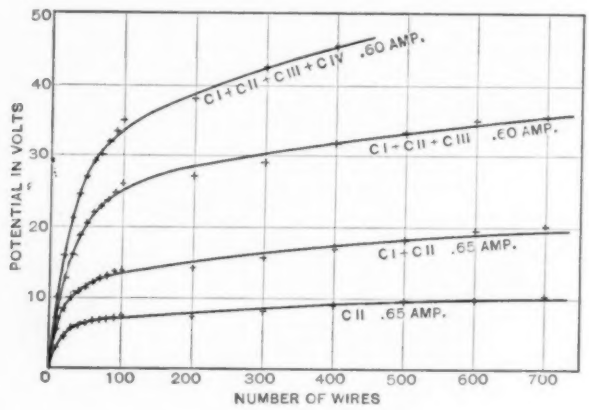

Fig. 6.

Table II. and Fig. 6 show how the form of the curve varies as the inductance in the secondary is increased by adding coils of the same number of turns and of the same diameter placed side by side.

TABLE II.

Number of Wires.	CII .65 Amp.	CI + CII .65 Amp.	CI + CII + CIII .60 Amp.	CI + CII + CIII + CIV .60 Amp.
	Volts.	Volts.	Volts.	Volts.
0	.0	.0	.0	.0
10	2.5	5.5	8.0	10.0
20	4.2	8.3	12.8	16.0
30	5.5	10.0	16.1	21.1
40	6.1	10.9	18.8	24.6
50	6.4	11.4	20.5	27.2
60	6.6	12.0	22.0	29.4
70	6.9	12.7	22.7	30.0
80	7.0	13.0	23.8	32.0
90	7.2	13.7	24.8	33.5
100	7.5	13.8	26.0	35.0
200	7.6	14.2	27.1	38.0
300	8.5	15.8	29.0	42.5
400	9.2	17.1	31.8	45.5
500	9.6	18.4	33.5	54.0
600	9.9	19.6	35.1	57.0
700	10.3	20.3	35.8	58.5

Table III. and Fig. 7 also show how the form of the curve varies as the inductance is increased, but in this case the increase

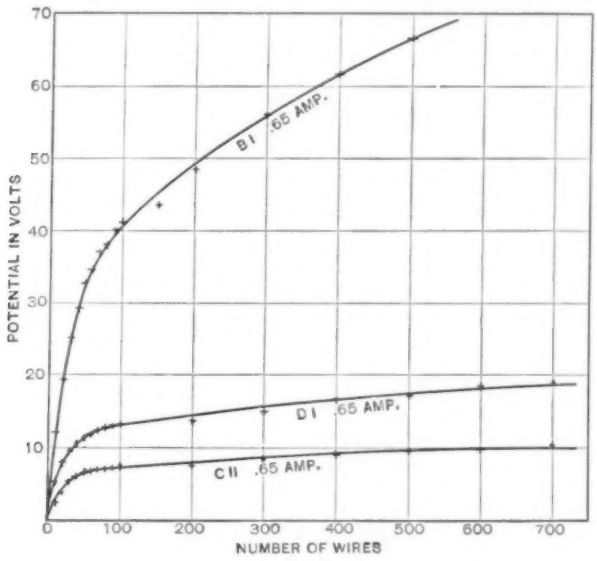

Fig. 7.

is made by replacing the original secondary by a coil of the same width, but of greater diameter and therefore of a greater number of turns.

TABLE III.

Number of Wires.	CII .65 Amp.	DI .65 Amp.	BI .65 Amp.
	Volts.	Volts.	Volts.
0	.0	.0	.0
10	2.5	5.3	12.2
20	4.2	8.0	19.2
30	5.5	9.7	25.2
40	6.1	10.5	29.3
50	6.4	11.4	32.6
60	6.6	11.8	34.5
70	6.9	12.3	37.0
80	7.0	12.7	37.8
90	7.2	12.9	40.0
100	7.5	13.2	40.7
200	7.6	13.7	48.4
300	8.5	15.1	56.0
400	9.2	16.6	62.0
500	9.6	17.3	66.8
600	9.9	18.5	68.0
700	10.3	19.0	

TABLE IV.

Number of Wires.	CII .65 Amp.	CII 1.05 Amp.	CII 2.3 Amp.
	Volts.	Volts.	Volts.
0	.0	.0	2.5
10	2.5	4.4	6.3
20	4.2	7.2	9.7
30	5.5	9.4	12.6
40	6.1	11.5	15.5
50	6.4	13.7	17.8
60	6.6	15.4	20.0
70	6.9	16.4	22.6
80	7.0	17.5	24.3
90	7.2	18.4	26.8
100	7.5	19.2	29.0
200	7.6	23.2	43.0
300	8.5	26.0	50.0
400	9.2	26.5	52.3
500	9.6	28.2	53.8
600	9.9	29.2	54.5
700	10.3	29.6	54.5

Tables IV., V. and Figs. 8, 9 show the change in the form of the curve for the same secondary coil as the current is increased.

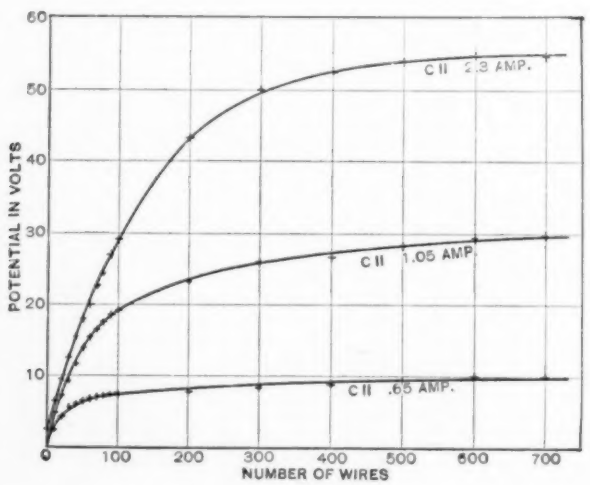

Fig. 8.

TABLE V.

Number of Wires.	DI .65 Amp.	DI 1.05 Amp.	Number of Wires.	DI .65 Amp.	DI 1.05 Amp.
	Volts.	Volts.		Volts.	Volts.
0	.0	.0	90	12.9	25.2
10	5.3	7.0	100	13.2	26.3
20	8.0	11.1	200	13.7	27.7
30	9.7	14.7	300	15.1	30.8
40	10.5	17.4	400	16.6	32.0
50	11.4	19.8	500	17.3	34.2
60	11.8	21.7	600	18.5	36.0
70	12.3	23.1	700	19.0	
80	12.7	24.3			

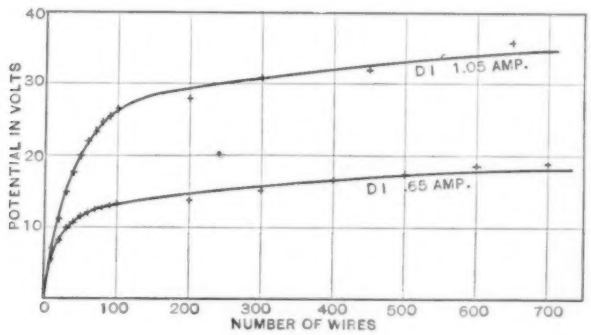

Fig. 9.

The form of these curves is given by equation (34).

Since M and L_1 are linear functions of the permeability of the core, and the permeability of air is very small compared with that of iron, we may assume that they are proportional to the number of iron wires in the core, and we may write

$$M = k_1 x,$$

$$L_1 = k_2 x,$$

where x is the number of iron wires in the core. Equation (34) then becomes

$$V_m = \frac{k_1 \omega E x}{\sqrt{k_2^2 \omega^2 x^2 + R_1^2}},$$

which is an equation of the fourth degree in V_m and x. The graph of V as a function of x gives a curve of the fourth order symmetrical with respect to both the x and y axes, and having points of inflexion at the origin. The portion of the curve plotted in the figures is for x positive and V positive. It is evident by inspection of equation (34) that when the number of wires is infinite V_m approaches a finite maximum value, $\frac{ME}{L_1}$. Also, that the potential increases very rapidly as the number of wires is increased up to a certain point after which an increase in the number of wires only makes a small difference in the potential. This is a fact of practical importance in the construction of coils.

The question arose as to whether it would make any difference whether the wires were placed together in the axis of the core, or placed loosely anywhere within the core. It was found by experiment that, within the limits of accuracy of the experiments, it made no difference whether the iron wires were placed in a bundle in the middle of the core or placed loosely anywhere within it.

§ 5. DISTRIBUTION OF THE INDUCTION.

A matter of considerable importance in the construction of coils is the knowledge of the flux of induction through the primary at different points of its length. It is important to know how the flux varies as we move along the coil from one end to the other, in order to know where to place the secondary coils so that the greatest flux shall pass through them.

To determine this, experiments were made, by placing upon the primary one of the secondary coils, and sliding it along from one end to the other. The primary was divided into ten equal parts, and the secondary was made to take eleven different positions at equal distances apart, including the ends. The terminals of the secondary coil were attached to the electrometer. The current used in the primary was the commercial alternating current, with a frequency of about 60 oscillations a second. Three different sized secondary coils were used, viz.: 5,000 turns, 2,500 turns and 1,250 turns. The coils used were BIII, DI, and CIV.

The results are given in Table VI. and Fig. 10.

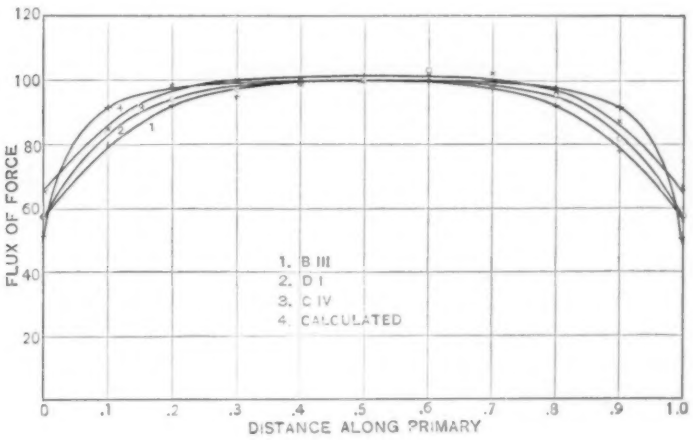

Fig. 10.

The method of performing the experiment was to move the secondary from left to right, noting the deflections for each of the eleven positions, and then move it back again from right to left noting the deflections, and finally take the mean of the two readings for the same position.

For BIII the mean primary current was 5.1 amp.; for DI, 5.6 amp.; for CIV, 8.4 amp.

TABLE VI.

Fraction of Total Length of Primary.	BIII		DI		CIV	
	Diff. of Pot. in Volts.	Reduced to 100.	Diff. of Pot. in Volts.	Reduced to 100.	Diff. of Pot. in Volts.	Reduced to 100.
.0	6.3	57	4.5	57	3.0	64
.1	8.8	80	6.6	84	4.0	85
.2	10.1	92	7.4	94	4.6	98
.3	10.5	95	7.7	97	4.7	100
.4	10.9	99	7.8	99	4.7	100
.5	11.1	101	7.9	100	4.7	100
.6	11.0	100	8.1	103	4.8	102
.7	10.8	98	7.8	99	4.8	102
.8	10.1	92	7.5	95	4.5	96
.9	8.6	78	6.5	82	4.1	87
1.0	6.2	56	4.6	58	3.1	66

The theory of this experiment is given by equation (34).

Since when we move the exploring coil along the primary everything is unchanged except M, we have V_m directly proportional to M. But M is directly proportional to the flux of induction through the solenoid at that point of its length. Therefore V_m is a measure of the flux of induction through the solenoid, and is directly proportional to it.

The flux of induction through the primary was also calculated. The variation of the field from that of an infinitely long solenoid is due to the free ends of the solenoid. The correction due to the ends was calculated from Maxwell's[1] formula for the force at any point due to a charged disk. The force was then integrated over a spherical cap, bounded by the solenoid, at different distances from the end of the solenoid. This gives the flux through the solenoid at different distances from its end. Using the first three terms of the series the following formula was obtained:

$$\text{Flux} = 4\pi^2 r^2 \left\{ \tfrac{1}{2} \frac{a^2}{r^2} \left(\frac{z}{r} - 1 \right) - \frac{3}{2.2.4} \frac{a^4}{r^4} \left[\left(\frac{z^3}{r^3} - 1 \right) - \left(\frac{z}{r} - 1 \right) \right] + \frac{3.5}{2.4.6.8} \frac{a^6}{r^6} \left[7 \left(\frac{z^5}{r^5} - 1 \right) - 10 \left(\frac{z^3}{r^3} - 1 \right) + 3 \left(\frac{z}{r} - 1 \right) \right] \right\},$$

Where a is the radius of the solenoid.

r the distance from the center of the end to the spherical cap.

z the distance from the end of that turn of the solenoid through which the cap passes.

This formula gives the flux through a turn of the solenoid at a distance z from the left hand end, due to a disc of unit density placed at the left hand end. To find the flux when unit current passes through the solenoid we must multiply by the number of turns in unit length of the coil. To get the total effect due to both ends, we must add the effects of the two ends, which are in the same direction. This gives the correction to be subtracted from the flux for an infinitely long solenoid. The solenoid used in these experiments was made of two layers, an inner and an outer layer. The flux was calculated for each separately and the total flux taken as the sum of the two. The results are given in Table VII. The results are only given for half the length of the coil, as the two

[1] Maxwell, Elect. & Mag., Vol. II., p. 284.

halves are, of course, symmetrical. The corrected flux through the solenoid is plotted as curve 4 in Fig. 10.

If the flux were calculated on the assumption that the primary was infinitely long, we should have 1,551 cm. for every point along the primary, instead of the values given in the column last but one of Table VII.

TABLE VII.

Frational distance from left hand end of coil.	Outer Coil.				Inner Coil.				Both Coils Together.		
	Flux due to disk of unit density at left hand end.	Flux due to disk of unit density at right hand end.	Sum of both $= S_1$	$S_1 \times 4.273 = F_1$	Flux due to disk of unit density at left hand end.	Flux due to disk of unit density at right hand end.	Sum of both $= S_2$	$S_2 \times 4.353 = F_2$	Total flux for unit current due to ends $= F_1 + F_2$	Total flux through primary.	Reduced to 100.
0	98.59	.39	98.98	422.9	83.35	.26	83.61	355.7	778.6	772	50.5
.1	19.54	.47	20.01	85.5	14.97	.34	15.31	65.1	150.6	1400	91.6
.2	7.48	.57	8.05	34.4	5.47	.40	5.87	25.0	59.4	1492	97.6
.3	3.72	.76	4.48	19.1	2.62	.56	3.18	13.6	32.7	1518	99.3
.4	2.21	1.00	3.21	13.7	1.54	.75	2.29	9.8	23.5	1527	99.9
.5	1.47	1.47	2.94	12.6	1.07	1.07	2.14	9.1	21.7	1529	100.0

The flux of induction through the primary at any point of its length is of course measured by the number of lines passing through a single turn wound tightly upon it at that point. This is the quantity calculated in Table VII. Since the coils BIII, CIV and DI, used in the experiment, consist of a large number of turns, most of the turns being situated at an appreciable distance from the primary, the flux through these coils will not be directly proportional to that for a single turn. This difference in the conditions may perhaps account for the difference between the curves for the observed and calculated values in Fig 10.

§ 6. CAPACITY IN THE SECONDARY.

Experiments were made to determine the effect of inserting localized capacity in series in the secondary.

The current used in the primary was the commercial alternating current. It was analyzed by Pupin's[1] method, and was found to contain the third and fifth harmonics.

[1] See Pupin, loc. c t.

For a given frequency, ω, the amplitude of the potential oscillation in the secondary condenser, is given by equation (33). If more than one frequency is present in the current, the potential is the sum of the potentials due to the different frequencies.

Experiments were made with each of the three frequencies present in the commercial current. The frequency desired was brought out by Pupin's method which has been described in § 3. To test the accuracy of Pupin's method the large resonance coil described in § 3 was placed in series with a condenser in a circuit which had impressed upon it the electromotive force of the commercial altering current. The fundamental of the alternating current was about 60 oscillations a second. The potential of the condenser was observed as its capacity was varied in the neighborhood of that capacity which would give resonance for the fundamental. These observations are given in Table VIII. The theoretical values were then calculated from equation (16). The results are given in Table IX. The observed potentials were relative, not absolute. The results are plotted in Fig. 11. The crosses represent the calculated values, and the circles the observed values. It is hard to say why the two curves do not coincide. If we should assume that the

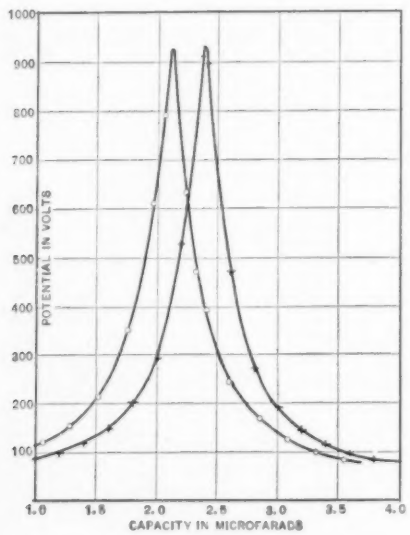

Fig. 11.

TABLE VIII.

Capacity in Microfarads.	Square Root of Deflection × 112.4.	Capacity in Microfarads.	Square Root of Deflection × 112.4.
1.06	120	2.32	470
1.26	151	2.40	393
1.51	212	2.60	248
1.76	352	2.86	167
1.97	612	3.09	126
2.06	793	3.30	101
2.23	634	3.55	82

TABLE IX.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
1.00	86	2.60	473
1.20	101	2.80	271
1.40	121	3.00	191
1.60	148	3.20	143
1.80	200	3.40	117
2.00	292	3.60	97
2.20	530	3.80	84
2.37	910	4.00	74
2.40	897		

large resonance coil has a distributed capacity of about .3 microfarad, the two curves would be very nearly coincident.

TABLE X.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
.000	17.1	.210	26.2	1.26	47.7
.032	17.7	.224	32.4	1.51	62.6
.038	18.0	.232	36.7	1.76	104.0
.052	18.3	.235	37.1	1.97	181.0
.059	18.4	.244	34.9	2.23	187.0
.071	19.2	.255	29.0	2.40	116.0
.080	21.9	.283	22.4	2.32	139.0
.083	24.1	.306	21.1	2.06	234.0
.090	25.0	.319	20.7	2.60	73.4
.095	22.1	.421	20.7	2.86	49.4
.107	19.5	.621	25.2	3.09	37.2
.127	19.0	.884	29.9	3.30	29.9
.155	19.4	1.060	35.5	3.55	24.3
.178	20.6				
.190	22.0				

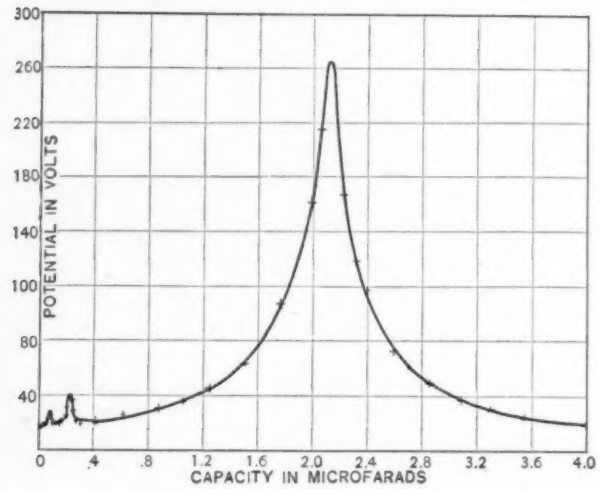

Fig. 12.

Table X. shows the results of the analysis of the commercial alternating current, made in the way described. The capacity of the circuit was gradually increased from nothing up to 3.5 microfarads. The observations are plotted in Fig. 12. The three peaks reading from right to left represent the fundamental, the third harmonic and the fifth harmonic, respectively.

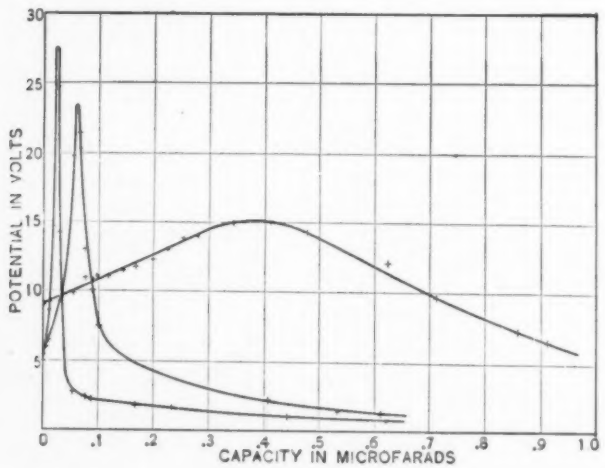

Fig. 13.

Having made these preliminary observations, the main problem was attacked, with the results given in Tables XI.–XIV., and Figs. 13–15.

TABLE XI.

BI + BIV.

Capacity in Microfarads.	Potential in Volts.		
	$n = 60$ 2.2 Amp. in Primary.	$n = 180$ .3 Amp. in Primary.	$n = 300$ .3 Amp. in Primary.
.000	9.1	5.0	4.6
.007	9.0	6.2	6.5
.008	9.2	6.0	6.7
.012	9.2	6.5	8.6
.019			14.7
.020			18.7
.027			24.7
.032	9.7	9.2	14.2
.060	9.9	19.7	2.8
.069		21.4	
.071		20.3	
.082	10.9	13.0	2.4
.095	10.8	10.0	2.2
.107	11.0	7.5	
.127	11.1		
.155	11.5		
.178	11.8		1.8
.210	12.3		
.238	13.0		
.250			1.6
.269	13.7		
.297	14.0		
.367	15.0		
.436	15.0	2.2	
.462			1.0
.506	14.2	1.7	
.659	12.0	1.2	.7
.755	9.5		
.909	7.1		
.968	6.3		

Tables XI. and XII. give the results obtained by changing the capacity of the secondary condenser, keeping the primary conditions constant. Three frequencies were used, viz.: those of the fundamental, the third overtone, and the fifth overtone. The results given in Table XI. were obtained by using BI and BIV in the

TABLE XII.

BI + BIV + BII.

Capacity in Microfarads.	Potential in Volts.		
	$n = 60$ 2.6 Amp. in Primary.	$n = 180$ .3 Amp. in Primary.	$n = 300$ .2 Amp. in Primary.
.000	12.0	6.8	7.0
.004		8.3	10.5
.007	12.0	9.7	15.0
.008		10.2	20.4
.012	12.4	13.0	32.5
.013			65.0
.016			27.3
.019			17.3
.020		15.4	13.0
.027		20.7	
.032	13.0	26.6	5.2
.038		28.1	
.040		26.0	
.043		22.5	
.060	14.3	9.9	1.5
.082	15.5	5.5	
.095	16.0	4.5	
.127	17.5		
.186	20.7		
.238	22.0		
.250	21.9		
.269	21.5		
.281	21.5		
.318	19.0		
.372	16.9		
.532	10.1		
.777	1.8		

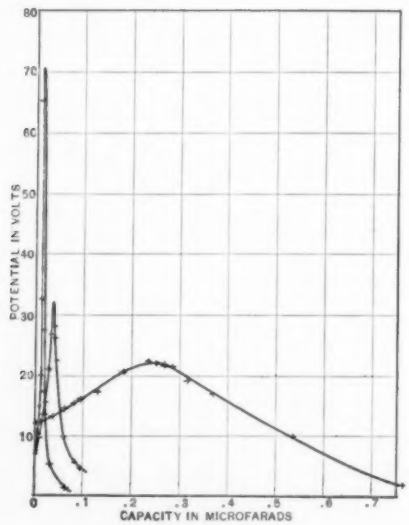

Fig. 14.

secondary, and those given in Table XII. by using BI, BIV and BII in the secondary. This gave a variation of the secondary inductance. BI + BIV and BI + BIV + BII have self inductances of 11.1 and 20.8 henries respectively. The observations are plotted in Figs. 13 and 14. The theory of these curves is given in the latter part of § 2, and the form of the resonance curves is given by equation (33).

Fig. 15 represents the "resonance curve" for the commercial current, without the weeding out of the overtones. The observations from which the curve was plotted are given in Table XIII. BI and BIV were used in the secondary, and the current in the primary was 2 amp. This curve shows, as already stated, that the commercial current contained two harmonics beside the fundamental. It leads to the same conclusion as Fig. 12, the difference being that in Fig. 12 the potential of the primary condenser is plotted, whereas in Fig. 15 the potential of the secondary condenser is plotted. Fig. 15 shows that a small induction coil and an electrometer form an excellent detector of harmonics.

TABLE XIII.

Capacity in Microfarads.	Potential in Volts.	Capacity in Microfarads.	Potential in Volts.
.000	16.1	.084	15.8
.026	43.8	.090	15.0
.030	30.9	.105	14.6
.031	27.8	.141	13.1
.033	25.5	.156	14.0
.038	19.7	.183	14.6
.039	18.6	.247	17.0
.044	17.4	.315	18.6
.046	17.2	.390	19.3
.050	16.3	.456	19.1
.053	18.0	.543	18.2
.057	18.1	.611	16.0
.063	18.8	.652	15.7
.068	18.8	.727	14.5
.074	18.0	.794	11.8

Figs. 13–15 show that the potential of the secondary in a small induction coil may in some cases be increased by the insertion of a condenser in series in the secondary.

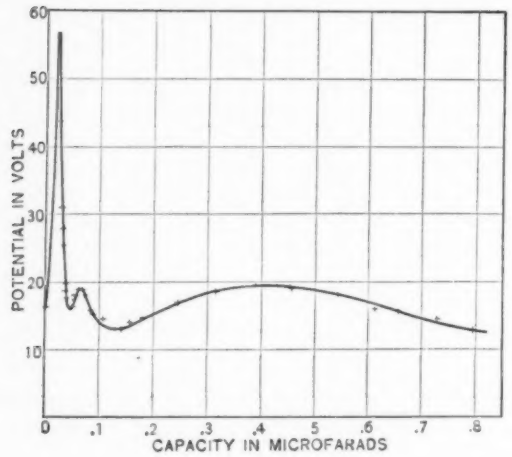

Fig. 15.

TABLE XIV.

Frequency.	Observed Value of K_m in Microfarads.	Calculated Value of K_m in Microfarads.
	$L_2 = 11.1$ henries.	
60	.403	.421
180	.066	.071
300	.024	.027
	$L_2 = 20.8$ henries.	
60	.237	.257
180	.036	.039
300	.014	.015

From these tables and curves can be found the value of the secondary capacity which will give the greatest difference of potential in the secondary for a given frequency. In Table XIV. are given the values read off from the curves, side by side with the values calculated from equation (35).

To be continued.

SURFACE COLOR.

By R. W. Wood.

So many misleading statements appear in the text-books on optics regarding the phenomenon of surface color, exhibited by certain strongly absorbing media, that it may be well to point out certain facts which are not always brought out as clearly as they should be. We frequently find the statement that these substances reflect most strongly those colors which are absent in the transmitted light, or which, as we more frequently say, are most strongly absorbed. One of the more recent of the advanced text-books refers the phenomenon to total reflection, the explanation being that for waves of certain wave-lengths the refractive index is less than one, and that therefore at a certain angle there will be total reflection for these waves, and consequent surface color. As a matter of fact the dispersion curve runs below unity only in the case of a very few substances, consequently total reflection is wholly inadequate as an explanation. Moreover the selective reflection occurs at normal incidence.

In the first place the statement that the most strongly reflected light corresponds to the light absorbed is erroneous. Take cyanine as an example. The center of the absorption band is very near the D lines, but the surface color is not yellow, but a deep plum color, not so very different in hue from that of the transmitted light. If we examine the spectrum of the reflected light we find a very dark band in the green, the center being not far from wave-length .0005. The distribution of intensity in the rest of the spectrum is not very different from what it would be in the case of reflection from glass, which shows that the peculiar color of the dye is not so much due to a very powerful reflection of certain waves, as it is to its almost complete refusal to reflect a certain region of the spectrum. If we remember that the reflecting power depends on the refractive index

as well as on the coefficient of absorption, we can easily explain this behavior.

The intensity of the light reflected at normal incidence by a transparent medium is given by the formula

$$R = \frac{(n-1)^2}{(n+1)^2}$$

As n, the refractive index, varies with the wave-length, the reflecting power will be different in different parts of the spectrum. The formula calls for a stronger reflecting power in the violet than in the red, the variation being greatest for substances with high dispersion and low mean refractive index. An experiment has recently been described by B. Walter to illustrate this type of selective reflection. Using the method of repeated reflections originated by Rubens and Nichols for the isolation of long heat waves, a remnant of blue violet light is obtained. The medium used by Walter was oil of cassia, above the surface of which a silver mirror was mounted.

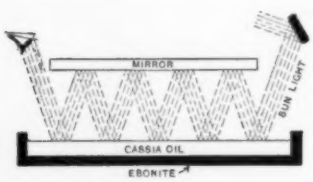

Fig. 1.

The sun's image after five reflections from the surface of the oil (the light being thrown back and forth between the surfaces as shown in Fig. 1), was found to be colored a deep blue. Though the formula calls for just such an effect, I have failed absolutely to obtain a trace of it though I employed six reflections, or one more than Walter. The image was very faint and of a neutral gray color with no suggestion of blue.

In the case of absorbing media the refractive index varies over a much wider range than it does in case of transparent substances, consequently greater variation in reflecting power with changing wave-length is to be expected. Moreover the absorption influences the reflecting power directly, the formula for media of this nature being

$$R = \frac{n^2 + n^2x^2 + 1 - 2n}{n^2 + n^2x^2 + 1 + 2n}.$$

In the case of a substance such as Hoffman's violet, the refractive index of which is unity for a certain wave-length, the amount of

light of this particular color reflected at normal incidence will be $\frac{\varkappa^2}{4+\varkappa^2}$. It is clear that this will increase as $\varkappa$ increases, and will equal zero if $\varkappa$ is equal to zero, as it is in the case of transparent media.

Taking the values of n and $\varkappa$ for cyanine obtained by Pflüger in the different parts of the spectrum, we can calculate the percentages reflected in the cases of the different colors. The curve shown in

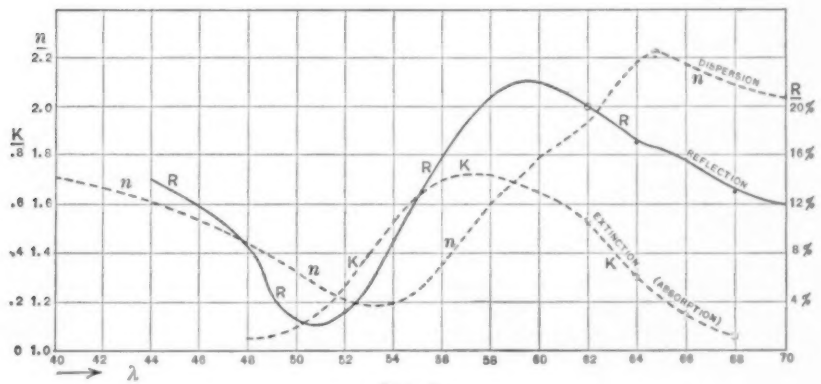

Fig. 2.

Fig. 2 was obtained in this way; it shows very clearly the cause of the peculiar surface color of the dye. Starting at the red end of the spectrum we find that the intensity of the reflected light increases with decreasing wave-length, due to the joint increase in n and $\varkappa$. The increase continues for some distance after n begins to decrease, owing to the continued increase of $\varkappa$. After reaching a maximum at $\lambda = .00059$ it turns and descends rapidly attaining its minimum at $\lambda = .0005$, where we have very small values of both n and $\varkappa$, and consequently very small reflecting power. Beyond this point the rise is due almost entirely to the further increase in n, since $\varkappa$ is practically equal to zero in this portion of the spectrum.

Scarcely two per cent. of the incident light is reflected at $\lambda = .0005$ causing the dark band seen in the spectrum of the reflected light, alluded to at the beginning of the paper.

An excellent way of showing the variable reflecting power of a film of cyanine is to compare it with glass, in different parts of the

spectrum. A little of the melted dye is pressed between two plates of hot glass, which are separated when cold. A spot is selected where the film has a good optical surface, and this spot is left on the glass the rest being cleaned off. By holding the plate in the spectrum formed by a prism or grating the reflecting power of the two surfaces can be studied. In some parts of the spectrum the cyanine reflects more strongly than glass, in other regions the reverse is true, while at wave-length .0005 the cyanine refuses to reflect to such a degree that the film appears as a black spot on the blue field reflected by the glass.

It is interesting to note the difference in the surface color of the dye when the reflection takes place at the surface in contact with the glass. A very convenient way of showing the yellowish-green color in this case is to press out a film of the molten dye on one surface of a prism of 8 or 10 degrees angle.[1] In this way the light reflected from the dye can be obtained uncontaminated with the light reflected from the first glass surface. The method is analogous to that employed by Lippmann in mounting his color photographs. The calculation of the curve of reflected intensities under these conditions makes a good exercise for the student.

[1] A suitable prism can be made in half an hour by grinding down a piece of thick plate window glass. A strip of thick glass cemented along one edge will be all that is necessary to make the glass take the required form. Grind on a piece of glass with very coarse emery at first, then use finer grades, polishing with rouge at the end. Small scratches do no harm, and a high polish is not necessary.

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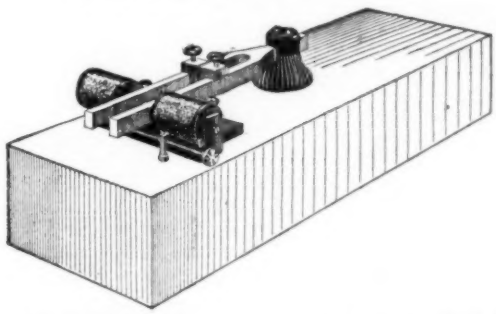

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